Quiz 4

Name: ____

1. Suppose z is given as a function of x and y by the equation $\sin(xyz)+x^2z^3-xy^4=3z$. Find:

a) $\dfrac{\partial z}{\partial x}$

Answer: Since z is given implicitly by $F(x,y,z)=0$, where $F(x,y,z)=\sin(xyz)+x^2z^3-xy^4-3z$, we can use the formula $\dfrac{\partial z}{\partial x}=\dfrac{-F_x}{F_z}$. This gives

$$\frac{\partial z}{\partial x}=\frac{-F_x}{F_z}=-\left(\frac{\cos(xyz)yz+2xz^3-y^4}{\cos(xyz)xy+3x^2z^2-3}\right).$$

b) $\dfrac{\partial z}{\partial y}$

Answer: Similarly to part a), we can use

$$\frac{\partial z}{\partial y}=\frac{-F_y}{F_z}=-\left(\frac{\cos(xyz)xz-4xy^3}{\cos(xyz)xy+3x^2z^2-3}\right).$$

2. Find the derivative of the function $f(x,y)=\dfrac{3}{x}+\log y$ at the point $(2,5)$ in the direction of the vector $\mathbf{v}=\langle 1,7\rangle$.

Answer: The unit vector in the direction of $\mathbf{v}$ is $\mathbf{u}=\dfrac{\mathbf{v}}{|\mathbf{v}|}=\dfrac{\langle 1,7\rangle}{\sqrt{50}}$, and the gradient of f at the point $(2,5)$ is $\nabla f=\langle f_x,f_y\rangle=\left\langle \dfrac{-3}{x^2},\dfrac{1}{y}\right\rangle=\left\langle \dfrac{-3}{4},\dfrac{1}{5}\right\rangle$. So

$$D_{\mathbf{u}}f=\nabla f\cdot\mathbf{u}=\left\langle \frac{-3}{4},\frac{1}{5}\right\rangle\cdot\frac{\langle 1,7\rangle}{\sqrt{50}}=\frac{-3}{4\sqrt{50}}+\frac{7}{5\sqrt{50}}.$$

3. Find the equation of the tangent plane to the surface $z^3+zx+z^2y^3=3$ at the point $(3,2,1)$.

Answer: the surface is a level curve of the function $F(x,y,z)=z^3+zx+z^2y^3$, so the tangent plane at $(3,2,1)$ is perpendicular to the gradient of F at $(3,2,1)$. This gradient vector is

$$\nabla F=\langle F_x,F_y,F_z\rangle=\langle z,3z^2y^2,3z^2+x+2zy^3\rangle=\langle 1,12,22\rangle.$$

So an equation of the tangent plane is $(x-3)+12(y-2)+22(z-1)=0$.