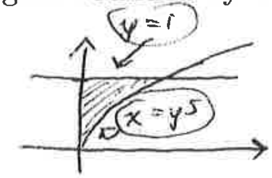
Quiz 5

Name: key

[10] **1.** Evaluate the double integral $\iint_R \frac{x}{1+y^{11}}\, dA$, where R is the region bounded by the curve $x = y^5$ and the lines $x = 0$ and $y = 1$ (see diagram).



(2) $\int_0^1 \int_0^{y^5} \frac{x}{1+y^{11}}\, dx\, dy$

$= \int_0^1 \left[\frac{x^2}{2} \cdot \frac{1}{1+y^{11}} \right]_{x=0}^{x=y^5} dy$ (2) $= \int_0^1 \frac{1}{2} y^{10} \cdot \frac{1}{1+y^{11}}\, dy$ (2)

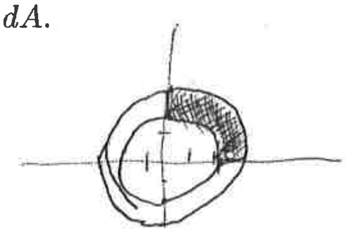
$u = 1+y^{11}$
$du = 11 y^{10}\, dy$
$y = 0 \to u = 1$
$y = 1 \to u = 2$

$= \int_1^2 \frac{1}{2} \frac{1}{1+y^{11}} \frac{11 y^{10}}{11}\, dy = \frac{1}{22} \int_1^2 \frac{1}{u}\, du$ (2) $= \frac{1}{22} \left[\ln u \right]_1^2$

$= \frac{1}{22} [\ln 2 - \ln 1] = \boxed{(\ln 2)/22.}$ (2)

2. The shaded region R in the diagram is the portion in the first quadrant of the ring between the circle centered at the origin of radius 2 and the circle centered at the origin of radius 3.

a) Use a double integral in polar coordinates to find $\iint_R x\, dA$.



$\iint x\, dA = \int_0^{\frac{\pi}{2}} \int_2^3 r \cos\theta$ (1) $r\, dr\, d\theta$ (1) (2)

$= \int_0^{\frac{\pi}{2}} \int_2^3 r^2 \cos\theta\, dr\, d\theta = \int_0^{\pi/2} \left[\frac{r^3}{3} \cos\theta \right]_2^3 d\theta$ (1)

$= \int_0^{\frac{\pi}{2}} \left[9 \cos\theta - \frac{8}{3} \cos\theta \right] = \int_0^{\frac{\pi}{2}} \frac{19}{3} \cos\theta\, d\theta$ (1) $= \frac{19}{3} \left[\sin\theta \right]_0^{\frac{\pi}{2}} = \frac{19}{3} \left(\sin\frac{\pi}{2} - \sin 0 \right)$

$= \boxed{\frac{19}{3}.}$ (1)

b) Find the x coordinate of the centroid of R, $\bar{x} = \dfrac{\iint_R x\, dA}{\iint_R dA}$.

$\iint_R dA = \int_0^{\frac{\pi}{2}} \int_2^3 r\, dr\, d\theta$ (1) $= \int_0^{\frac{\pi}{2}} \left[\frac{r^2}{2} \right]_2^3 d\theta$

$= \int_0^{\frac{\pi}{2}} \left(\frac{9}{2} - 2 \right) d\theta = \int_0^{\frac{\pi}{2}} 5/2\, d\theta = \frac{5\pi}{4}.$ (1)

So $\bar{x} = \dfrac{(19/3)}{(5\pi/4)} = \boxed{\dfrac{76\pi}{15}.}$ (1)