**Quiz 6**

Name: Key

**1.** Evaluate the line integral $\int_C x^2y\,dx + xy^2\,dy$

*a)* when $C$ is the curve given by $x = t$ and $y = t^2$ for $0 \le t \le 1$.

[5] (1) $\frac{dx}{dt} = 1$ and $\frac{dy}{dt} = 2t$, so the integral is

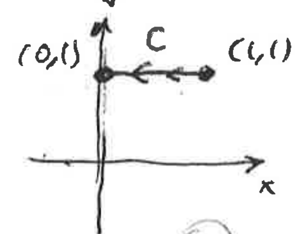
$$\int_0^1 \left[t^2\cdot t^2\cdot 1 + t\cdot(t^2)^2\, 2t\right]dt \quad (2)$$

$$= \int_0^1 \left[t^4 + 2t^6\right]dt = \left[\frac{t^5}{5} + \frac{2t^7}{7}\right]_0^1 \quad (1)$$

$$= \frac{1}{5} + \frac{2}{7} = \boxed{\frac{17}{35}} \quad (1)$$

*b)* when $C$ is the line segment which starts at $(1,1)$ and ends at $(0,1)$.

[5] Parametrize $C$ by $\begin{cases} x = 1-t \\ y = 1 \end{cases}$ for $0 \le t \le 1$. (1)




(other parametrizations will work as well.) Then

$\frac{dx}{dt} = -1$, $\frac{dy}{dt} = 0$ (1), so the integral is (1)

$$\int_0^1 \left[(1-t)^2\cdot 1\cdot(-1) + (1-t)1^2\cdot 0\right]dt \quad (1)$$

$$= -\int_0^1 (1-t)^2\,dt = -\int_0^1 (1+t^2-2t)\,dt = -\left[t + \frac{t^3}{3} - t^2\right]_0^1$$

$$= \boxed{-\frac{1}{3}}. \quad (1)$$

**2.** Suppose $\mathbf{F}(x,y,z) = (y^2 + 2xz)\mathbf{i} + (2xy + 2y)\mathbf{j} + x^2\mathbf{k}$.

*a)* Find a function $f$ such that $\nabla f = \mathbf{F}$.

$\frac{\partial f}{\partial x} = y^2 + 2xz$ (1) $\Rightarrow f = xy^2 + x^2z + g(y,z)$ (1)

$\Rightarrow \cancel{2xy} + 2y = \frac{\partial f}{\partial y} = \cancel{2xy} + 0 + \frac{\partial g}{\partial y}$ $\Rightarrow 2y = \frac{\partial g}{\partial y}$ (1) $\Rightarrow g = y^2 + h(z)$ (1)

$\Rightarrow f = xy^2 + x^2z + y^2 + h(z)$. Then $\cancel{x^2} = \frac{\partial f}{\partial z} = 0 + \cancel{x^2} + 0 + h'(z)$,

so $h'(z) = 0$, so $h(z) = C$ (1). So $f = \boxed{xy^2 + x^2z + y^2 + C}$ (1)

*b)* Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $C$ is any curve which starts at $(0,0,0)$ and ends at $(1,1,1)$.

By The Fundamental Theorem of Line Integrals,

$$\int_C \vec{F}\cdot d\vec{r} = f(1,1,1) - f(0,0,0) = (1+1+1+C) - (0+0+0+C)$$

(2)

$$= \boxed{3.} \quad (2)$$