Quiz 7

Name: key

1. If $\mathbf{F} = x^2\mathbf{i} + yz\mathbf{j} + xz^2\mathbf{k}$, find

[3] *a)* div $\mathbf{F}$ $= \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(yz) + \frac{\partial}{\partial z}(xz^2)$

$= \boxed{2x + z + 2xz}$

[3] *b)* curl $\mathbf{F}$ $= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & yz & xz^2 \end{vmatrix} = (0-y)\vec{i} - (z^2-0)\vec{j} + 0\,\vec{k}$

$= \boxed{-y\vec{i} - z^2\vec{j}}$

2. Suppose S is the portion of the surface $z = x^2 + y^2$ lying directly above the square $0 \le x \le 1$, $0 \le y \le 1$ in the xy-plane.

a) Parameterize the surface S, giving x, y, and z as functions of two parameters u and v.

2 $\begin{cases} x = u \\ y = v \\ z = u^2 + v^2 \end{cases}$

b) Find $\mathbf{r}_u$, $\mathbf{r}_v$, and $\mathbf{r}_u \times \mathbf{r}_v$

3 $\vec{r}_u = \langle \frac{dx}{du}, \frac{dy}{du}, \frac{dz}{du} \rangle = \langle 1, 0, 2u \rangle$

$\vec{r}_v = \langle \frac{dx}{dv}, \frac{dy}{dv}, \frac{dz}{dv} \rangle = \langle 0, 1, 2v \rangle$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2u \\ 0 & 1 & 2v \end{vmatrix} = -2u\vec{i} - 2v\vec{j} + 1\cdot\vec{k}$

c) Give an equation of the tangent plane to S at the point $(\frac{1}{2}, \frac{1}{3}, \frac{13}{36})$.

3 $\vec{r}_u \times \vec{r}_v = \langle -2\cdot\frac{1}{2}, -2\cdot\frac{1}{3}, +1 \rangle = \langle -1, -\frac{2}{3}, +1 \rangle$ at this point, so an equation is $(-1)(x-\frac{1}{2}) - \frac{2}{3}(y-\frac{1}{3}) + (z - \frac{13}{16}) = 0$.

d) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = 2xy\mathbf{i} + z\mathbf{k}$.

$$\iint_S \vec{F}\cdot d\vec{S} = \int_0^1\int_0^1 \langle 2xy, 0, z \rangle \cdot \langle -2u, -2v, +1 \rangle \, du\, dv \quad (1)$$

$$= \int_0^1\int_0^1 \langle 2uv, 0, u^2+v^2 \rangle \cdot \langle -2u, -2v, +1 \rangle \, du\, dv \quad (1)$$

$$= \int_0^1\int_0^1 \left[-4u^2v + u^2 + v^2\right] du\, dv \quad (1) \quad = \int_0^1 \left[-\frac{4u^3}{3}v + \frac{u^3}{3} + uv^2\right]_0^1 dv \quad (1)$$

$$= \int_0^1 \left[-\frac{4}{3}v + \frac{1}{3} + v^2\right] dv = \left[-\frac{2v^2}{3} + \frac{v}{3} + \frac{v^3}{3}\right]_0^1 \quad (1) \quad = -\frac{2}{3} + \frac{1}{3} + \frac{1}{3} = 0. \quad (1)$$