Math 2934 (Honors) — Fall 2015
Review for first exam

The first exam will be on the material in Chapter 12 and Sections 13.1 and 13.2 of the text. This material was covered on Assignments 1 through 3.

12.1. Three-dimensional coordinate systems. This section contains background material on how equations in three variables correspond to geometric objects in space. A basic rule of thumb (with some exceptions) is that in three-dimensional space with coordinates x, y, and z, one equation between the coordinates defines a two-dimensional surface; two equations define a one-dimensional curve; and three equations define a point. The underlying idea is that the dimension of an object corresponds to the number of independent variables (or "degrees of freedom") that it takes to describe the object, and each additional equation reduces the number of independent variables by one. Thus, if you describe an object by a single equation in x, y, and z, then there are really only two independent variables (because you can choose x and y freely, but then the equation determines the value of z for you) — so the object is two-dimensional.

In Example 2 on page 812 you see that the equation $x^2 + y^2 = 1$ determines a two-dimensional cylinder; while the two equations $x^2 + y^2 = 1$ and $z = 3$ together determine a one-dimensional circle. Another important example is the equation of a sphere (see Examples 5 and 6).

12.2. Vectors. For us a "vector in $\mathbf{R}^3$" is defined as a triple of numbers (called "components" of the vector) taken in a certain order. A vector with components a_1, a_2, and a_3 is denoted by the symbol $\langle a_1, a_2, a_3\rangle$. Notice the "angles" on either side: in our textbook, these are reserved for vectors.

So a vector looks a lot like a point in space, which has three coordinates. In our textbook points in space with coordinates x, y, z are written with parentheses: (x, y, z).

You might then ask: what's the difference between a vector, such as the vector $\langle 2, -1, 3\rangle$, and a point in space, such as the point $(2, -1, 3)$? For us, the answer is that we can do algebraic operations on vectors: you can add two vectors, or you can multiply a vector by a number, or you can take the dot product of two vectors, or you can take the cross product of two vectors. (There are other interesting operations on vectors as well — see for example the Wikipedia article on "outer product".) By contrast, we don't do algebraic operations on points.

One important use of a vector is to describe a *displacement* in space. This is just a fancy word for "where one point Q is relative to another point P". Given two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in space, we picture the displacement from P to Q as an arrow which starts at P and ends at Q, and we represent this displacement by the vector $\langle a_1, a_2, a_3\rangle$, where $a_1 = x_2 - x_1$, $a_2 = y_2 - y_1$, and $a_3 = z_2 - z_1$. This vector is called $\overrightarrow{PQ}$. Notice that you can have two completely different points R and S such that the vector $\overrightarrow{RS}$ is the same as the vector $\overrightarrow{PQ}$, in the sense that $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ have the same three components. For example, for $P(0, 2, 5)$ and $Q(1, 1, 7)$, we have $\overrightarrow{PQ} = \langle 1, -1, 2\rangle$; and for $R(11, 6, 4)$ and $S(12, 5, 6)$ we have $\overrightarrow{RS} = \langle 1, -1, 2\rangle$; so $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are actually the same vector. Thus displacements are determined by the placements of two points relative to each other; not by the actual positions of the points in space.

An important bit of terminology is the phrase "position vector of a point". If $P(x, y, z)$ is a point in space, then the *position vector* of this point is the displacement vector $\overrightarrow{OP}$ from the origin $O(0, 0, 0)$ to the point P. It is usually denoted in this book by the symbol $\mathbf{r}$. Thus the position vector of $P(3, 1, -5)$ is the vector $\mathbf{r} = \overrightarrow{OP} = \langle 3, 1, -5\rangle$. Notice that the components of $\mathbf{r}$ are the same as the coordinates of P. This makes it easy to confuse a point with its position vector. But you have to carefully distinguish between the two when reading the book, or things will quickly stop making sense!

There are also vectors in $\mathbf{R}^2$, which are defined similarly but are arrows between two points in the plane. In fact, vectors turn out to be a useful notion in any number of dimensions; though in this class we only talk about vectors in two or three dimensions.

You should review this entire section through Example 6. So you should be familiar with the notions of length or magnitude of a vector, and with addition of vectors and multiplication of vectors by scalars. You do not need to know the material in the subsection titled "Applications" on pages 821–22.

12.3. The dot product. You should review the entire section, except you can skip the subsection titled "Direction angles and direction cosines" on pages 827–828, and you do not need to know the material about work on page 829 (Examples 7 and 8).

12.4. The cross product. You should review the entire section, except you can skip the subsection titled "Torque" at the end of the section.

12.5. Equations of lines and planes. You should review the entire section, except you can skip the material on distances between lines and planes in Examples 8, 9, and 10.

12.6. Equations of quadric surfaces. You should review Examples 1 through 6. You do not need to memorize the information in Table 1 on page 854.

13.1. Vector functions and space curves. There is not much new in this section. A "vector function" is simply a vector whose components are functions of some variable (typically we use the letter t for this variable). Thus it has the form $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, where $f(t)$, $g(t)$, and $h(t)$ are functions of t. If we specify that $\mathbf{r}(t)$ be drawn as an arrow with its tail at the origin, then the tip of the arrow $\mathbf{r}(t)$ will trace out a curve in space as t varies. This section just consists of a few examples of vector functions together with drawings of the curves they trace out.

13.2. Derivatives and integrals of vector functions. In this section, it is explained (among other things) that if $\mathbf{r}(t)$ is a vector function tracing out a curve in space, then at any given point on the curve corresponding to a given value of t, the derivative $\mathbf{r}'(t)$ of the vector function (defined in the boxes on pages 871 and 872) defines a vector which is tangent to the curve.

You should review the entire section.

Note: I also mentioned in class that if $\mathbf{r}(t)$ is the position vector of a moving particle in space, then the velocity vector of the moving particle is $\mathbf{r}'(t)$, the speed of the particle is $|\mathbf{r}'(t)|$, and the acceleration is $\mathbf{r}''(t)$. This material is discussed in section 13.4 of the text (roughly speaking I covered in class the material on pages 886–888 in section 13.4). I won't ask about this material on the first exam, but you should know it anyway; it gives you a better understanding of vector functions and their derivatives, and I will return to it later in the course.