Quiz 4

Name: key

1. Suppose $f(x,y) = xy^2$.

[3] *a)* Find ∇f.

$$\vec{\nabla} f = f_x \vec{i} + f_y \vec{j} = y^2 \vec{i} + 2xy \vec{j} = \boxed{\langle y^2, 2xy \rangle}$$

[5] *b)* Find the directional derivative of f at the point $(1,3)$ in the direction of the vector $\langle 2,5 \rangle$.

A unit vector in this direction is $\vec{u} = \frac{\langle 2,5\rangle}{|\langle 2,5\rangle|} = \frac{\langle 2,5 \rangle}{\sqrt{29}}$. 2

The directional derivative is

$$D_{\vec{u}} f(1,3) = \underbrace{\vec{\nabla} f(1,3)}_{2} \cdot \vec{u} = \langle 9,6 \rangle \cdot \frac{\langle 2,5\rangle}{\sqrt{29}} = \underbrace{\frac{9\cdot 2 + 6 \cdot 5}{\sqrt{29}}}_{1} = \boxed{\frac{48}{\sqrt{29}}}$$

2. Suppose $F(x,y,z) = x^2 - 5x + z^3 + z - xyz$.

a) Find ∇F.

[3]

$$\vec{\nabla} F = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$= \boxed{(2x - 5 - yz)\vec{i} + (-xz)\vec{j} + (3z^2 + 1 - xy)\vec{k}}$$

[5] *b)* Find the equation of the tangent plane to the surface $F(x,y,z) = -2$ at the point $(1,1,1)$.

The plane is normal to $\vec{\nabla} F(1,1,1) = (2-5-1)\vec{i} - \vec{j} + (3+1-1)\vec{k}$

$= \langle -4, -1, 3 \rangle$, 2

so it has equation $\boxed{(-4)(x-1) + (-1)(y-1) + 3(z-1) = 0}$ 3

3. Find the critical point of the function $f(x,y) = xy - 3x + 2y$, showing all work.

[4] The critical point is the point where $f_x = 0$ and $f_y = 0$.

Now $f_x = 0 \Rightarrow y - 3 = 0 \Rightarrow y = 3$, and 2

$f_y = 0 \Rightarrow x + 2 = 0 \Rightarrow x = -2$.

So the critical point is $\boxed{(-2,3)}$. 2