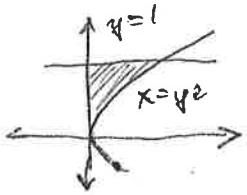
Quiz 5

Name: key

1. The region D is bounded by the lines $x = 0$ and $y = 1$ and the curve $x = y^2$ (see figure).



a. [6] Evaluate $\int\int_D x\,dx\,dy$, showing all work.

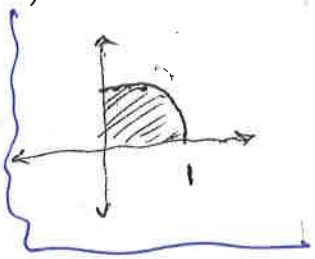
$$\int_0^1 \int_0^{y^2} x\,dx\,dy = \int_0^1 \left[\frac{x^2}{2}\right]_{x=0}^{x=y^2} dy = \int_0^1 \left[\frac{y^4}{2} - 0\right] dy = \left[\frac{y^5}{10}\right]_0^1 = \boxed{\frac{1}{10}}$$

b. [6] Evaluate $\int\int_D x\,dy\,dx$, showing all work.

$$\int_0^1 \int_{\sqrt{x}}^1 x\,dy\,dx = \int_0^1 \left[xy\right]_{y=\sqrt{x}}^{y=1} dx = \int_0^1 \left[x - x\sqrt{x}\right] dx = \int_0^1 \left[x - x^{3/2}\right] dx =$$

$$= \left[\frac{x^2}{2} - \frac{2}{5}x^{5/2}\right]_0^1 = \frac{1}{2} - \frac{2}{5} = \boxed{\frac{1}{10}}$$

2. [8] Let D be the quarter circle of radius 1 in the first quadrant (see figure). Evaluate $\int\int_D x\,dA$, using polar coordinates, and showing all work.



$$\int_0^{\frac{\pi}{2}} \int_0^1 r\cos\theta \; r\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \int_0^1 r^2\cos\theta\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{r^3}{3}\cos\theta\right]_{r=0}^{r=1} d\theta = \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{3}\,d\theta =$$

$$= \left[\frac{\sin\theta}{3}\right]_0^{\frac{\pi}{2}} = \frac{\sin\frac{\pi}{2}}{3} - \frac{\sin 0}{3} = \frac{1}{3}$$