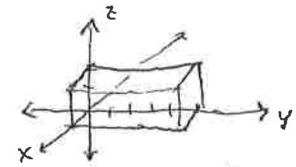
Quiz 6

Name: key

[10] **1.** The solid B is bounded by the the coordinate planes and the planes $x = 1$, $y = 5$, and $z = 2$ (see figure). Evaluate $\displaystyle\iiint_B (2x + 3y)\, dV$.



2

$$\int_0^2 \int_0^5 \int_0^1 (2x+3y)\, dx\, dy\, dz =$$

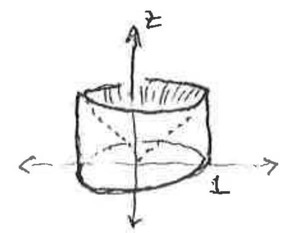
2   2

$$= \int_0^2 \int_0^5 \Big[x^2 + 3xy \Big]_{x=0}^{x=1} dy\, dz = \int_0^2 \int_0^5 (1+3y)\, dy\, dz =$$

$$= \int_0^2 \Big[y + \frac{3y^2}{2} \Big]_{y=0}^{y=5} dz = \int_0^2 \Big[5 + \frac{75}{2} \Big] dz = 2 \cdot \Big(5 + \frac{75}{2} \Big) = \boxed{85}$$

2   2

[10] **2.** The solid E lies above the xy-plane, within the unit ~~circle~~ cylinder $r = 1$, and below the cone $z = r$ (see figure). Evaluate $\displaystyle\iiint_E z^2\, dV$.



2   2

$$\int_0^{2\pi} \int_0^1 \int_0^r z^2\, r\, dz\, dr\, d\theta =$$

1

$$= \int_0^{2\pi} \int_0^1 \Big[\frac{z^3 r}{3} \Big]_{z=0}^{z=r} dr\, d\theta =$$

2   2

$$= \int_0^{2\pi} \int_0^1 \frac{r^4}{3}\, dr\, d\theta = \int_0^{2\pi} \Big[\frac{r^5}{15} \Big]_{r=0}^{r=1} d\theta$$

$$= \int_0^{2\pi} \frac{1}{15}\, d\theta = \boxed{\frac{2\pi}{15}}$$

1