Math 2934 — Summer 2015
Review for first exam

The first exam will be on the material in Chapter 12 and Sections 13.1 and 13.2 of the text. This material was covered on Assignments 1 through 4.

12.1. Three-dimensional coordinate systems. This section contains background material on how equations in three variables correspond to geometric objects in space. A basic rule of thumb (with some exceptions) is that in three-dimensional space with coordinates x, y, and z, one equation between the coordinates defines a two-dimensional surface; two equations define a one-dimensional curve; and three equations define a point. In Example 2 on page 812 you see that the equation $x^2 + y^2 = 1$ determines a two-dimensional cylinder; while the two equations $x^2 + y^2 = 1$ and $z = 3$ together determine a circle. Another important example is the equation of a sphere (see Examples 5 and 6).

12.2. Vectors. For us a "vector in $\mathbf{R}^3$" is defined as an arrow which starts at one point $P(x_1, y_1, z_1)$ in space and ends at another point $Q(x_2, y_2, z_2)$. This vector is called $\overrightarrow{PQ}$, and is also denoted by the symbol $\langle a_1, a_2, a_3\rangle$, where $a_1 = x_2 - x_1$, $a_2 = y_2 - y_1$, and $a_3 = z_2 - z_1$. The numbers a_1, a_2, and a_3 are called the x-component, y-component, and z-component of the vector. It is understood that if R and S are another pair of points and the vector $\overrightarrow{RS}$ turns out to have the same three components as $\overrightarrow{PQ}$ then $\overrightarrow{RS}$ and $\overrightarrow{PQ}$ are actually the same vector; there is no distinction between them. That is, two vectors can actually be the same vector even if they have different starting points, provided they have the same three components.

There are also vectors in $\mathbf{R}^2$, which are defined similarly but are arrows between two points in the plane. In fact, vectors turn out to be a useful notion in any number of dimensions; though in this class we only talk about vectors in two or three dimensions.

You should review this entire section through Example 6. So you should be familiar with the notions of length or magnitude of a vector, and with addition of vectors and multiplication of vectors by scalars. You do not need to know the material in the subsection titled "Applications" on pages 821–22.

12.3. The dot product. You should review the entire section, except you can skip the subsection titled "Direction angles and direction cosines" on pages 827–828, and you do not need to know the material about work on page 829 (Examples 7 and 8).

12.4. The cross product. You should review the entire section, except you can skip the subsection titled "Torque" at the end of the section.

12.5. Equations of lines and planes. You should review the entire section, except you can skip the material on distances between lines and planes in Examples 8, 9, and 10.

12.6. Equations of quadric surfaces. You should review Examples 1 through 6. You do not need to memorize the information in Table 1 on page 854.

13.1. Vector functions and space curves. There is not much new in this section. A "vector function" is simply a vector whose components are functions of some variable (typically we use the letter t for this variable). Thus it has the form $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$, where $f(t)$, $g(t)$, and $h(t)$ are functions of t. If we specify that $\mathbf{r}(t)$ be drawn as an arrow with its tail at the origin, then the tip of the arrow $\mathbf{r}(t)$ will trace out a curve in space as t varies. This section just consists of a few examples of vector functions together with drawings of the curves they trace out.

13.2. Derivatives and integrals of vector functions. In this section, it is explained (among other things) that if $\mathbf{r}(t)$ is a vector function tracing out a curve in space, then at any given point on the curve corresponding to a given value of t, the derivative $\mathbf{r}'(t)$ of the vector function (defined in the boxes on pages 871 and 872) defines a vector which is tangent to the curve.

You should review the entire section.

Note: I also mentioned in class that if $\mathbf{r}(t)$ is the position vector of a moving particle in space, then the velocity vector of the moving particle is $\mathbf{r}'(t)$, the speed of the particle is $|\mathbf{r}'(t)|$, and the acceleration is $\mathbf{r}''(t)$. This material is discussed in section 13.4 of the text (roughly speaking I covered in class the material on pages 886–888 in section 13.4). I won't ask about this material on the first exam, but you should know it anyway; it gives you a better understanding of vector functions and their derivatives, and I will return to it later in the course.