Quiz 1

Name: key

[7] **1.** Find the solution of $\frac{dy}{dx} = \frac{x}{(x^2+4)^2}$ which satisfies $y(1) = 0$.

$$y = \int \frac{x}{(x^2+4)^2}\,dx = \frac{1}{2}\int \frac{du}{u^2} = \frac{1}{2}\int u^{-2}\,du = \frac{1}{2}\frac{u^{-1}}{-1} + C$$

$u = x^2+4$, $du = 2x\,dx$

So $y = \frac{-1}{2(x^2+4)} + C$

since $y = 0$ when $x = 1$, then $0 = \frac{-1}{2\cdot 5} + C$, so $C = \frac{1}{10}$,

and $y = \frac{-1}{2(x^2+4)} + \frac{1}{10}$

[7] **2.** Find the solution of $\frac{dy}{dx} = \frac{3x^2+4x}{2y}$ which satisfies $y(1) = 2$. Give y explicitly as a function of x.

$$\int 2y\,dy = \int (3x^2+4x)\,dx \Rightarrow y^2 = x^3 + 2x^2 + C$$

Since $y = 2$ when $x = 1$, $4 = 1 + 2 + C \Rightarrow C = 1$

So $y^2 = x^3 + 2x^2 + 1$, $y = \sqrt{x^3+2x^2+1}$

3. In a city whose population P satisfies $\frac{dP}{dt} = kP$, the population is 10,000 in year $t = 0$ and is 20,000 in year $t = 10$. Find the value of k.

$P = De^{kt}$, and $P = 10{,}000$ at $t = 0$

$\Rightarrow 10{,}000 = D \cdot e^0$

$\Rightarrow D = 10{,}000$

$\Rightarrow P = 10{,}000\,e^{kt}$

Since $P = 20{,}000$ when $t = 10$,

$20{,}000 = 10{,}000\,e^{k\cdot 10}$, so

$2 = e^{10k}$, so $10k = \ln 2$, $k = \frac{\ln 2}{10}$