Quiz 2

Name: Key

[7] **1.** For the exact equation $(3x^2 + 4y\sin x\cos x) + (2\sin^2 x + y^3)y' = 0$, find the general solution (you can give the solution in implicit form).

(The equation $M + N\frac{dy}{dx} = 0$ is exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Here $M = 3x^2 + 4y\sin x\cos x$ and $N = 2\sin^2 x + y^3$, so $\frac{\partial M}{\partial y} = 4\sin x\cos x = \frac{\partial N}{\partial x}$, so the equation is exact.)

We have $F = \int^x (3x^2 + 4y\sin x\cos x) = x^3 + 4y\int \sin x\cos x\,dx = x^3 + 4y\int u\,du =$ (1) (1) (1)

$u = \sin x$, $du = \cos x\,dx$ (1)

$= x^3 + 4y\cdot\frac{u^2}{2} + g(y) = x^3 + 2y\sin^2 x + g(y)$. (1)

and $\frac{\partial F}{\partial y} = 0 + 2\sin^2 x + g'(y) = N = 2\sin^2 x + y^3$, so $g'(y) = y^3$, $g(y) = \frac{y^4}{4} + C$. So (1)

$F(x,y) = x^3 + 2y\sin^2 x + \frac{y^4}{4} + C$, (1) and a general solution is given implicitly by $\boxed{x^3 + 2y\sin^2 x + \frac{y^4}{4} = D}$ where D is an arbitrary constant

[5] **2.** Use the Wronskian to show that the functions $\sin x$ and $\cos x$ are linearly independent.

For $y_1 = \sin x$ and $y_2 = \cos x$, we have (1)

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -\sin^2 x - \cos^2 x = -1.$$

(1) (1) (1)

So $W(y_1, y_2) \neq 0$ for all x, so $y_1 = \sin x$ and $y_2 = \cos x$ are linearly independent (1)

[8] **3.** Find the solution of the initial-value problem $y'' - 7y' + 12y = 0$ with initial condition $y(0) = 1$ and $y'(0) = 1$.

The characteristic equation is $r^2 - 7r + 12 = 0$, (1) which factors as $(r-3)(r-4) = 0$, (1) with real and unequal roots $r = 4$ and $r = 3$,

so the general solution is $y = C_1 e^{3x} + C_2 e^{4x}$. (1)

Then we have $y' = 3C_1 e^{3x} + 4C_2 e^{4x}$ (1)

So $y(0) = 1 \Rightarrow C_1 + C_2 = 1$ (1)

$y'(0) = 1 \Rightarrow 3C_1 + 4C_2 = 1$ (1)

$$\Rightarrow \begin{array}{r} 3C_1 + 3C_2 = 3 \\ -(3C_1 + 4C_2 = 1) \\ \hline -C_2 = 2 \end{array} \Rightarrow C_2 = -2$$

$\Rightarrow C_1 = 1 - C_2 = 1 - (-2) = 3$. (1) So $\boxed{y = 3e^{3x} - 2e^{4x}}$ (1)