Quiz 3

Name: key

**1.** Use the method of undetermined coefficients to find the general solution of $y'' - 3y' = 4x$.

[9] The characteristic equation is $r^2 - 3r = 0$, or $(r-3)r = 0$, 1

with roots $r = 0$ and $r = 3$, so the associated homogeneous 1

equation has general solution $y_c = C_1 e^{0x} + C_2 e^{3x} = C_1 + C_2 e^{3x}$.

For $y_p$ we should try $y_p = x(Ax+B) = Ax^2 + Bx$. 2

Then $y_p' = 2Ax + B$ and $y_p'' = 2A$, so $y_p'' - 3y_p' = 4x \Rightarrow$ 1

$2A - 3(2Ax+B) = 4x \Rightarrow -6Ax + (2A - 3B) = 4x \Rightarrow$

$$\begin{cases} -6A = 4 \\ (2A-3B) = 0 \end{cases} \text{ 1 } \Rightarrow A = -\tfrac{2}{3} \text{ and } B = \tfrac{2A}{3} = \text{ 1 } -\tfrac{4}{9}. \text{ So } y_p = -\frac{2x^2}{3} - \frac{4}{9}x \text{ 1}$$

and $y = y_c + y_p = \boxed{C_1 + C_2 e^{3x} - \frac{2x^2}{3} - \frac{4x}{9}}$ 1

**2.** Find $\mathcal{L}^{-1}\left\{\dfrac{3s+2}{s^2+49}\right\}$.

[5]

$$3\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+7^2}\right\} + \frac{2}{7}\,\mathcal{L}^{-1}\left\{\frac{7}{s^2+7^2}\right\} = \boxed{3\cos(7t) + \frac{2}{7}\sin(7t)}$$

1 1 3

[6] **3.** If $x'' + 3x' + 2x = 1$, $x(0) = 4$, and $x'(0) = 5$, find the Laplace transform $X(s)$ of $x(t)$. (You do not need to find $x(t)$ itself.)

1 $\left[s^2 X(s) - s\cdot x(0) - x'(0)\right] + 3\left[s X(s) - x(0)\right] + 2X(s) = \frac{1}{s}$, 1

so $(s^2 + 3s + 2) X(s) - 4s - 5 - 3.4 = \frac{1}{s}$, 1 1

and $(s^2 + 3s + 2) X(s) = \frac{1}{s} + 4s + 17$,

$$\boxed{X(s) = \frac{1}{s(s^2+3s+2)} + \frac{4s}{(s^2+3s+2)} + \frac{17}{(s^2+3s+2)}}$$ 2