EXAM 1
Math 4103
2-25-11

Name key

1. *(15 points)* Let $z = \dfrac{8-7i}{(3i)(4+2i)}$. Find

a. Re z $\underline{\frac{-44}{60}}$ $\left(= \frac{-11}{15}\right)$

b. Im z $\underline{\frac{-18}{60}}$ $\left(= \frac{-9}{30}\right) = \left(\frac{-3}{10}\right)$

$$z = \frac{(8-7i)(4-2i)}{3i(4^2+2^2)} = \frac{(8\cdot 4 - 2\cdot 7) - i(4\cdot 7 + 2\cdot 8)}{60i}$$ 4 4 4

$$= \frac{18 - 44i}{60i}$$

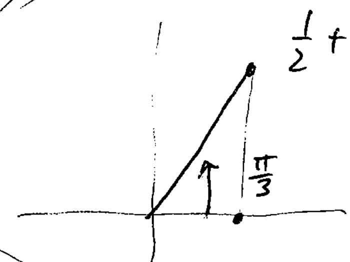
$$= \frac{-44}{60} - \frac{18}{60}i$$ 3

2. *(15 points)* Let $z = \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{47}$. Find

a. Arg z $\underline{-\pi/3}$

b. Re z $\underline{1/2}$

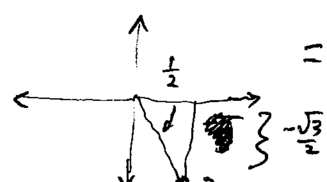
c. Im z $\underline{-\sqrt{3}/2}$



$$\frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{i\pi/3}$$ 5

$$z = \left(e^{i\pi/3}\right)^{47} = e^{i\frac{47\pi}{3}} =$$ 5

$$= e^{16\pi i} e^{-\pi i/3} = e^{-\pi i/3}$$



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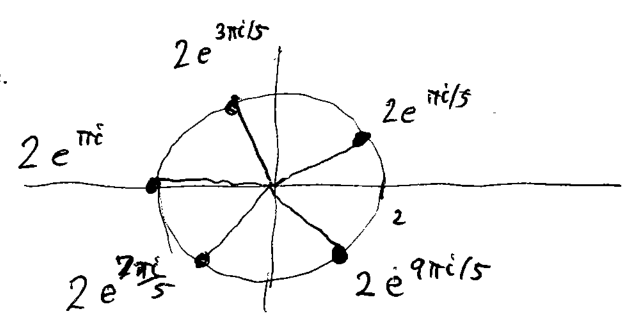
3. *(20 points)* The number $z = -32$ has one real 5^{th} root: namely, -2. However, it has a total of five complex 5^{th} roots.

$$z = 32e^{\pi i} = 32e^{3\pi i} = 32e^{5\pi i} = 32e^{7\pi i} = 32e^{9\pi i}$$

a. Find all five values of $(-32)^{1/5}$ (in exponential form) $\left\{2e^{\pi i/5}, 2e^{3\pi i/5}, 2e^{\pi i}, 2e^{\frac{7\pi i}{5}}, 2e^{\frac{9\pi i}{5}}\right\}$ 12

b. Of the five values of in part a, which is the principal root? $\underline{2e^{\pi i/5}}$ 2

c. Graph the five 5^{th} roots in the complex plane. 4



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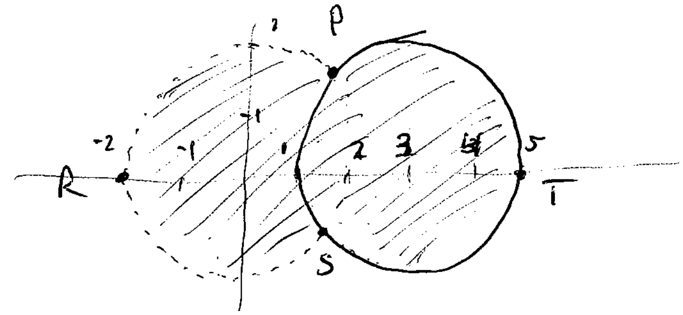
4. *(10 points)* Let S be the set consisting of all points z such that $|z| < 2$ or $|z-3| \le 2$.

a. Sketch the set S:

(2)



Answer the following questions, giving brief reasons for your answers:

b. What are the boundary points of S?

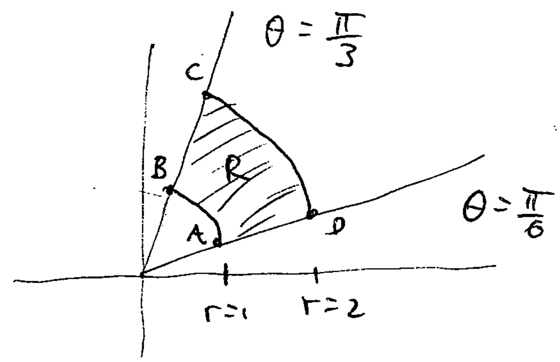
(2) {The arc PRS of the circle $|z|=2$} $\cup$ {the arc PTS of the circle $|z-3|=2$}

(2) **c.** Is S open? no, because it contains some of its boundary points (e.g. $z=5$)

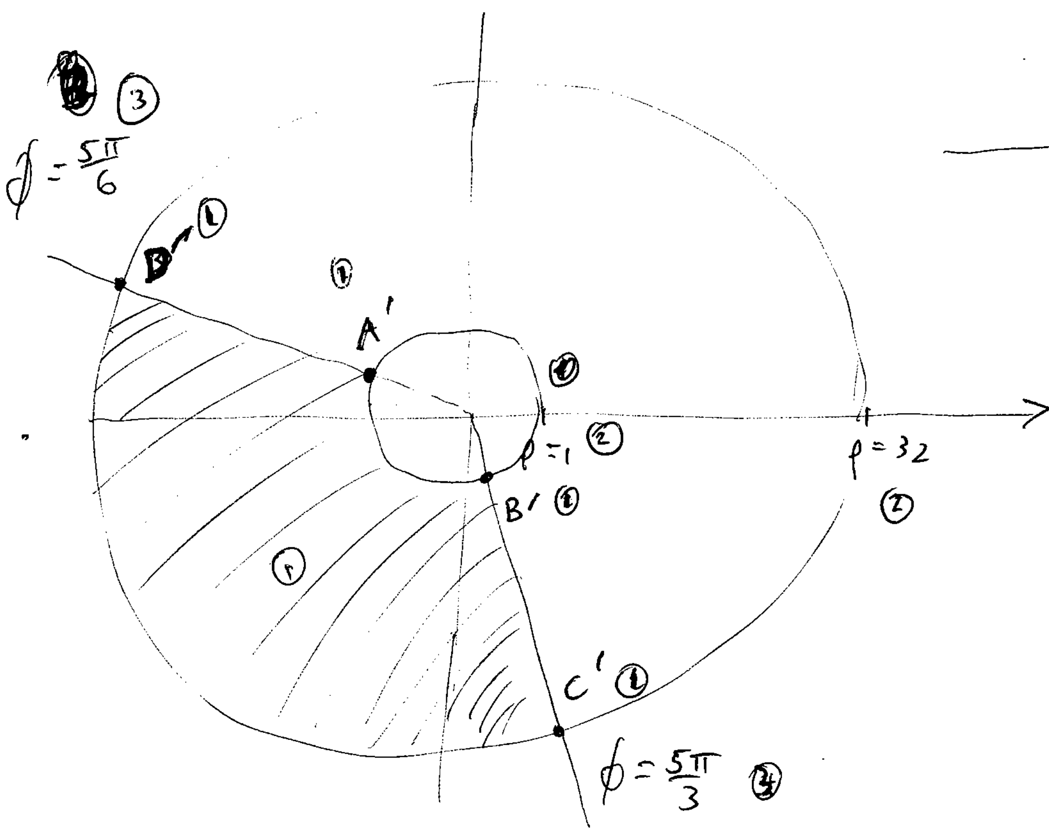
(2) **d.** Is S connected? yes, any two points in S can be connected by a polygonal path in S

(2) **e.** Is S a domain? no, because it is not open.

5. *(15 points)* Let $w = z^5$, and let R be the region in the z-plane shown in the diagram below. Sketch the image of R in the w-plane. Label the ρ and ϕ values of the boundaries, and label the images of the points A, B, C, D.



(3)



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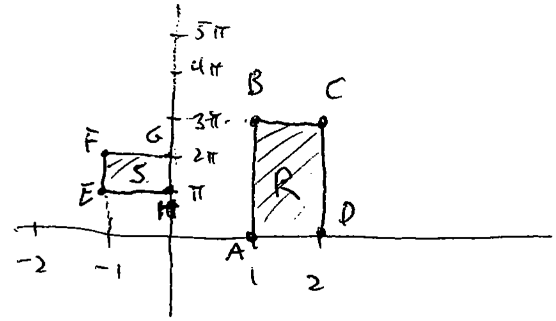
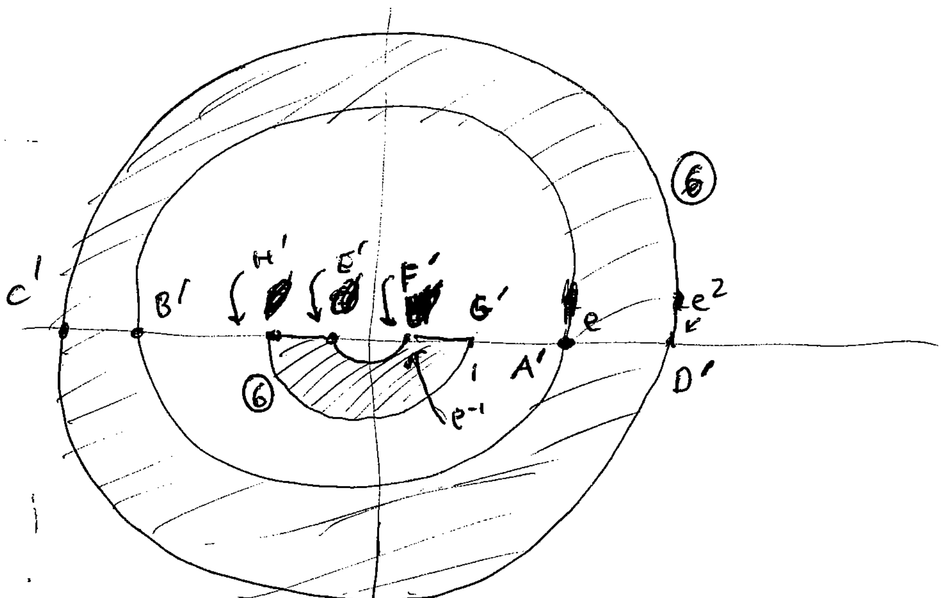
6. *(15 points)* Let $w = e^z$, and let R and S be the regions in the z-plane shown in the diagram below.

a. Sketch the images of R and S in the w-plane, labeling the images of the points A, B, C, D, E, F, G, H.





b. Can you find two different numbers z_1 and z_2 in R which are mapped to the same point? If so, exhibit them.

2 Yes, for example $z_1 = 1$ and $z_2 = 1 + 2\pi i$: $e^{z_1} = e^{z_2} = e$

c. Can you find two different numbers z_1 and z_2 in S which are mapped to the same point? If so, exhibit them.

1 No.

7. *(10 points)* Find all complex numbers z such that $\bar{z} = z^2$. (Hint: first take the modulus of both sides.)

$$\bar{z} = z^2 \Rightarrow |\bar{z}| = |z^2| \Rightarrow |z| = |z|^2 \Rightarrow \begin{cases} |z| = 0 & 1 \\ \text{or} \\ |z| = 1 & 5 \end{cases}$$

– If $|z| = 0$, Then $z = 0$

– If $|z| = 1$, Then $z = e^{i\theta}$, so $\bar{z} = e^{-i\theta} = \frac{1}{z}$,

and $\bar{z} = z^2 \Rightarrow \frac{1}{z} = z^2 \Rightarrow 1 = z^3$ 2 $\Rightarrow$

$\Rightarrow z = 1^{1/3} = \{e^{i\frac{2\pi}{3}}, e^{i\frac{4\pi}{3}}, e^{i\frac{6\pi}{3}}\} = \{e^{\frac{i2\pi}{3}}, e^{\frac{i4\pi}{3}}, 1\}$. 2

Answer: $\{0, 1, e^{\frac{2\pi i}{3}}, e^{\frac{4\pi i}{3}}\}$.