# Review for Second Exam

The second exam will cover sections 15 through 26 of the text. The problems should be similar to those on homework assignments numbers 5, 6, and 7. Here is a brief discussion of what parts of the sections we covered in class.

**15. Limits.** This section contains a definition of limit in terms of $\epsilon$'s and $\delta$'s, which one would have to use in order to prove theorems about limits. In this class we take a more intuitive approach, and take the basic theorems about limits on faith, rather than proving them. I won't ask questions on the exam which require knowing this $\epsilon$-$\delta$ definition. However, it is good to keep in mind that to settle any question about limits definitively, one must ultimately return to this definition.

One of the more or less intuitive results about limits that we took on faith was that, in order for $\lim_{z \to z_0} f(z)$ to exist, the value of $f(z)$ must approach the same limit as $z$ approaches $z_0$ in a horizontal direction as when $z$ approaches $z_0$ in a vertical direction (see example 2 on page 47). This is a necessary, but not a sufficient, condition for the existence of the limit at $z_0$.

**16. Theorems on limits.** You can skip Theorem 1 and its proof. The material from Theorem 2 through to the end of the section is worth reviewing.

**17. Limits involving the point at infinity.** We covered this entire section.

**18. Continuity.** All that you might need to know from this section is the definition of continuity (see the first sentence of the section) and the statement of Theorem 1. You do not need to know the proof of Theorem 1. You can skip Theorems 2 and 3.

**19. Derivatives.** Everything in this section is important. The word "differentiable" is defined in this section: saying a function is "differentiable" at a point means that the derivative exists, in the sense that the limit which defines the derivative exists.

**20. Differentiation formulas.** This section just establishes that the usual rules of differentiation for real-valued functions also hold for complex derivatives. You will not need to know the proofs in this section. It's worth noting, though, that all the proofs here are exactly the same as the proofs of the differentiation rules for real-valued functions.

**21. Cauchy-Riemann equations.** The heart of this section is the theorem on page 65, which gives a necessary but not sufficient condition for a function to be differentiable at a point. The proof of this theorem occupies pages 63 and 64; I will not ask you for this proof, but I think it is a good idea to have at least a rough idea of how the proof goes. It is related to the "more or less intuitive" fact about limits that I referred to above in commenting on section 15. Thinking about this relationship will help you tie the ideas of the course together.

At any rate, you should have the Cauchy-Riemann equations memorized by the time you take the test.

**22. Sufficient conditions for differentiability.** The theorem in the preceding section gave necessary (but not sufficient) conditions for differentiability of a function at a point. The theorem in this section gives sufficient (but not necessary) conditions. Compare the two theorems carefully. To understand their relationship, it might help to note that exercise 6 on page 72 shows that the theorem in section 21 does not give sufficient conditions for differentiability, while the example of $f(z) = |z|^2$ at $z = 0$ shows that the theorem in section 22 does not give necessary conditions for differentiability.

**23. Polar coordinates.** This section restates the theorem from section 21 in terms of polar coordinates. I won't ask for a proof of the theorem. I won't expect you to know the proof of this theorem, but you should probably try to memorize the polar form of the Cauchy-Riemann equations (see (6) on page 69), if for no other reason than to free up your brain for other things on the exam.

**24. Analytic functions.** We covered the entire section. The theorem on page 74 is useful to know, but you won't need to know its proof.

**25. Examples.** All worth reading.

**26. Harmonic functions.** We covered the entire section. The theorem on page 79 is one of the rare theorems for which I think you should learn the proof, which is both short and instructive.