Math 4103
Review for Final Exam

The final exam is comprehensive, so will cover all the material covered on the first three exams, plus some extra material covered between the third exam and the end of class. To review for the final, in conjunction with the review sheets for the first three exams you can review the following material from Chapter 5 of the text. The relevant assignment for the material discussed below is Assignment 11.

55. Convergence of sequences. This section gives the rigorous definition of limit of a sequence, which you can skip. You should have already seen limits of sequences of real numbers in calculus, and have some intuitive idea of what they are. For sequences of complex numbers, the idea is the same. Namely, if $z_1, z_2, z_3, \ldots$ is a sequence of complex numbers, we use the phrase $\lim_{n\to\infty} z_n = z$ to mean that the numbers in the sequence can be made arbitrarily close to the fixed number z by going out far enough in the sequence, or in other words by taking n sufficiently large.

56. Convergence of series. The notion of convergence of a series should also be something you have seen before in calculus, but it's probably worth reviewing it carefully here since it's so fundamental to the theory of complex variables. The basic definition is in the first paragraph of this section, which you should understand clearly. You can skip the following theorem and its corollaries 1 and 2, if you like, but you should understand the material in the rest of the section, starting from the last paragraph on the bottom of page 186 and going through the following example, which is fundamental.

57. Taylor's theorem. This section is short but important, and should be reviewed in detail. Notice in particular the second paragraph on page 190, beginning "When it is known that ... ". This goes along with the paragraph on page 216 that begins with "We observed in Sec. 57 that ... ". The point of these paragraphs is that you can find the radius of convergence of a Taylor series merely by looking at the points where the corresponding function fails to be analytic. There is no need to use convergence tests or remainder estimates of the type used for real-valued functions in your calculus class. I gave a couple of examples illustrating this on the last day of class.

58. Proof of Taylor's theorem. Instead of going through the details of this proof, I gave a shortened version in class. I think it would help your understanding of the material in general to review the proof I gave in class, as it ties together Taylor's theorem, the Cauchy integral formula, and the formula for the sum of a geometric series (that is, the example on page 187). You can skip the more detailed version in the text.

59. Examples. The examples and exercises in this section are like some of the problems you can expect to see on the final. When reading these examples it will no doubt occur to you that it's a good idea to memorize some of the basic Taylor series, such as the ones for e^z, $\sin z$, $\cos z$, and $1/(1-z)$.

60. Laurent's theorem. 61. Proof of Laurent's theorem. By the time we got to these sections, we did not have much time left in the class, so I did not cover them in much detail. Instead I merely stated Laurent's theorem (the theorem on pages 197-8) and discussed it briefly before going on to a couple of examples like the ones in section 62. You can just briefly review the statement of the theorem and skip the rest of these sections.

62. Examples. All the examples in this section are worth reviewing for the exam. You can also look at problems 11 to 13 on page 197, which are similar. Don't overlook the fact that part of doing these problems is to find the correct values of z for which the series converge.

Other material. You will not need to know any of the material in the remaining sections of Chapter 5 for the final exam. The same goes for the material I discussed on the last day of class from Chapter 6, concerning residues. All the contour integrals that I might ask for in class can be found using the methods from Chapter 4 that you learned for the third exam. However, as I mentioned in class, knowing the residue theorem (page 235) and a simple formula for finding residues (Theorem 2 on page 253) can potentially be quite useful, since together they give you a simpler way to compute many integrals.