Math 4163
Assignment 4

1. In this problem you'll find three different series for what looks like the same function.

(a) Let $f(x) = x + 1$. Find constants B_n such that

$$f(x) = \sum_{n=1}^{\infty} B_n \sin(n\pi x)$$

for $0 < x < 1$.

(b) Let $f(x) = x + 1$. Find constants A_n such that

$$f(x) = \sum_{n=0}^{\infty} A_n \cos(n\pi x)$$

for $0 < x < 1$.

(c) Let $f(x) = x + 1$. Find constants A_n and B_n such that

$$f(x) = A_0 + \sum_{n=1}^{\infty} [A_n \cos(n\pi x) + B_n \sin(n\pi x)]$$

for $-1 < x < 1$.

2. Use the method of separation of variables to find a solution of the problem

$$\begin{aligned}
&(1) \quad u_{tt} = u_{xx} \quad \text{for } 0 < x < 1,\ t > 0 \\
&(2) \quad u(0,t) = 0 \quad \text{for } t > 0 \\
&(3) \quad u(1,t) = 0 \quad \text{for } t > 0 \\
&(4) \quad u(x,0) = f(x) \quad \text{for } 0 \le x \le 1 \\
&(5) \quad u_t(x,0) = g(x) \quad \text{for } 0 \le x \le 1.
\end{aligned}$$

Step 1: Put $u(x,t) = \phi(x)G(t)$ and write down the ordinary differential equations satisfied by $\phi(x)$ and $G(t)$. The equation for $\phi(x)$ should be the same as it was for the heat equation; the equation for $G(t)$ will be slightly different.

Step 2a: Find the boundary conditions implied for $\phi(x)$ by conditions (2) and (3) above. Write down the eigenvalues and eigenfunctions for this boundary-value problem for $\phi(x)$. (This step will be the same as for the heat equation. You can just use the results we obtained before, you don't have to derive them again.)

Step 2b: Find the general solution of the ordinary differential equation for $G(t)$. This step will be slightly different than for the heat equation; there will be two arbitrary constants.

Completion of Step 2: Write down the functions of the form $u(x,t) = \phi(x)G(t)$ you have found which satisfy (1), (2), and (3) above. These are the "separated solutions" of (1), (2), and (3).

Step 3: Set $u(x,t)$ equal to a linear combination of all the separated solutions you found in Step 2. (This will be a series involving two sequences of arbitrary constants.) Use (4) to find formulas for some of these constants as integrals involving $f(x)$. Then write down a formula for $u_t(x,t)$, and use (5) to find formulas for the remaining constants as integrals involving $g(x)$. This will complete your solution of (1), (2), (3), (4), and (5).