Math 4163
Assignment 5

1. Find the solution of the problem

$$\begin{aligned} u_{xx} + u_{yy} &= 0 && \text{for } 0 < x < 2 \text{ and } 0 < y < 3, \\ u_x(0,y) &= 0 && \text{for } 0 \le y \le 3, \\ u_x(2,y) &= 0 && \text{for } 0 \le y \le 3, \\ u(x,0) &= 0 && \text{for } 0 \le x \le 2, \\ u(x,3) &= f(x) && \text{for } 0 \le x \le 2, \end{aligned}$$

where

$$f(x) = \begin{cases} 0 & \text{for } 0 \le x \le 1 \\ 1 & \text{for } 1 < x \le 2. \end{cases}$$

(You can use the solution to the eigenvalue problem for $\phi(x)$ from the inside front cover of the book without having to rederive it here.) Your answer will be in the form of a series for $u(x,y)$. The coefficients in the series will be determined by the function f.

2. Consider the problem of finding the solution $u(x,y)$ of

$$\begin{aligned} u_{xx} + u_{yy} &= 0 && \text{for } 0 < x < 1 \text{ and } 0 < y < 1, \\ u_x(0,y) &= 0 && \text{for } 0 \le y \le 1, \\ u_x(1,y) &= 0 && \text{for } 0 \le y \le 1, \\ u_y(x,0) &= 0 && \text{for } 0 \le x \le 1, \\ u_y(x,1) &= f(x) && \text{for } 0 \le x \le 1, \end{aligned}$$

where f is a function given by

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\pi x)$$

for $0 \le x \le 1$.

(a) Use the method of separation of variables to show that u will be of the form

$$u(x,y) = P_0 + Q_0 y + \sum_{n=1}^{\infty} (P_n \cosh(n\pi y) + Q_n \sinh(n\pi y)) \cos(n\pi x).$$

Explain how you know that $Q_n = 0$ for $n = 0, 1, 2, \ldots$.

(b) Use the boundary condition at $y = 1$ to find expressions for the unknown constants P_n in terms of the given constants A_n. In particular, show that if A_0 is not zero, then the problem does not have a solution; but on the other hand, if A_0 does equal zero, then the problem does have solutions, but does not have a unique solution.