Math 4163
Assignment 7

Note: In solving these problems, if there are any steps which have already been worked out in the class lectures, you can use what we did in class without having to repeat the derivations.

1. Find the solution $u(x, y, t)$ of the following boundary-value problem for the heat equation on the rectangle $\mathcal{D} = \{(x, y) : 0 < x < L,\ 0 < y < H\}$.

$$\begin{aligned}
u_t &= k(u_{xx} + u_{yy}) && \text{for } (x, y) \in \mathcal{D} \text{ and } t > 0,\\
u_x(0, y, t) &= 0 && \text{for } 0 \le y \le H \text{ and } t \ge 0,\\
u_x(L, y, t) &= 0 && \text{for } 0 \le y \le H \text{ and } t \ge 0,\\
u_y(x, 0, t) &= 0 && \text{for } 0 \le x \le L \text{ and } t \ge 0,\\
u_y(x, H, t) &= 0 && \text{for } 0 \le x \le L \text{ and } t \ge 0,\\
u(x, y, 0) &= \alpha(x, y) && \text{for } (x, y) \in \mathcal{D}.
\end{aligned}$$

Here $\alpha(x, y)$, the initial temperature distribution within the rectangle, is a known function. Your answer will be a series for u with coefficients which are given in terms of integrals involving $\alpha(x, y)$.

2. Find the solution $u(x, y, z, t)$ of the following boundary-value problem for the heat equation on the box $\mathcal{D} = \{(x, y, z) : 0 < x < L,\ 0 < y < H,\ 0 < z < W\}$.

$$\begin{aligned}
u_t &= k(u_{xx} + u_{yy} + u_{zz}) && \text{for } (x, y, z) \in \mathcal{D} \text{ and } t > 0,\\
u(0, y, z, t) &= 0 && \text{for } 0 \le y \le H,\ 0 \le z \le W, \text{ and } t \ge 0,\\
u(L, y, z, t) &= 0 && \text{for } 0 \le y \le H,\ 0 \le z \le W, \text{ and } t \ge 0,\\
u_y(x, 0, z, t) &= 0 && \text{for } 0 \le x \le L,\ 0 \le z \le W, \text{ and } t \ge 0,\\
u_y(x, H, z, t) &= 0 && \text{for } 0 \le x \le L,\ 0 \le z \le W, \text{ and } t \ge 0,\\
u_z(x, y, 0, t) &= 0 && \text{for } 0 \le x \le L,\ 0 \le y \le H, \text{ and } t \ge 0,\\
u(x, y, W, t) &= 0 && \text{for } 0 \le x \le L,\ 0 \le y \le H, \text{ and } t \ge 0,\\
u(x, y, z, 0) &= \alpha(x, y, z) && \text{for } (x, y, z) \in \mathcal{D}.
\end{aligned}$$

Here $\alpha(x, y, z)$, the initial temperature distribution within the box, is a known function. Your answer will be a series for u with coefficients which are given in terms of integrals involving $\alpha(x, y, z)$.

3. Find the solution $u(x, y, z)$ of the following boundary-value problem for Laplace's equation on the box $\mathcal{D} = \{(x, y, z) : 0 < x < L,\ 0 < y < H,\ 0 < z < W\}$.

$$\begin{aligned}
u_{xx} + u_{yy} + u_{zz} &= 0 && \text{for } (x, y, z) \in \mathcal{D},\\
u_x(0, y, z) &= 0 && \text{for } 0 \le y \le H \text{ and } 0 \le z \le W\\
u_x(L, y, z) &= 0 && \text{for } 0 \le y \le H \text{ and } 0 \le z \le W\\
u(x, 0, z) &= 0 && \text{for } 0 \le x \le L,\ 0 \le z \le W\\
u(x, H, z) &= 0 && \text{for } 0 \le x \le L,\ 0 \le z \le W\\
u(x, y, 0) &= 0 && \text{for } 0 \le x \le L \text{ and } 0 \le y \le H\\
u(x, y, W) &= \alpha(x, y) && \text{for } 0 \le x \le L \text{ and } 0 \le y \le H,
\end{aligned}$$

Here $\alpha(x, y)$ is a known function. Your answer will be a series for u with coefficients which are given in terms of integrals involving $\alpha(x, y)$.