EXAM 2 Name ______________________________
Math 4163
3-29-13

Instructions *Work all of the following problems in the space provided. If there is not enough room, you may write on the back sides of the pages. Give thorough explanations to receive full credit.*

1. *(25 points)* Solve Laplace's equation in polar coordinates,

$$\frac{1}{r}\left(\frac{\partial u}{\partial r}+r\frac{\partial^2 u}{\partial r^2}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0,$$

for $u(r,\theta)$ on the semicircle of radius 2 given by $0 \le r \le 2$ and $0 \le \theta \le \pi$, with the boundary conditions

$$u(r,0)=0 \quad \text{and} \quad u(r,\pi)=0$$

for $0 \le r \le 2$, and

$$\frac{\partial u}{\partial r}(2,\theta)=f(\theta)$$

for $0 \le \theta \le \pi$, where $f(\theta)$ is a given function. You may assume that $u(r,\theta)$ does not go to infinity as r approaches zero.

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2. *(15 points)* The function $f(x) = \begin{cases} 1 & 0 \leq x \leq \pi/2 \\ 3 & \pi/2 < x \leq \pi \end{cases}$ is graphed at right.

a) Sketch the Fourier cosine series of $f(x)$ for $-3\pi \leq x \leq 3\pi$, marking with $\times$'s the value of the series at each discontinuities.

b) Find the coefficients of the Fourier cosine series for $f(x)$.

3. *(10 points)* For the equation

$$\frac{1}{r}\left(\frac{\partial u}{\partial r} + r\frac{\partial^2 u}{\partial r^2}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + 3\frac{\partial u}{\partial \theta} = 0,$$

write $u(r, \theta) = \phi(\theta)G(r)$ and separate the variables to obtain two ordinary differential equations for $\phi(\theta)$ and $G(r)$. (Just write down these differential equations; you need not solve them.)

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4. *(25 points)* Consider the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},$$

for $-L < x < L$ and $t > 0$.

a) Find all solutions of the form $u(x,t) = \phi(x)h(t)$ of the equation, subject to the boundary conditions

$$u(-L,t) = u(L,t) \quad \text{and} \quad \frac{\partial u}{\partial x}(-L,t) = \frac{\partial u}{\partial x}(L,t).$$

b) One of the solutions $u(x,t) = \phi(x)h(t)$ in part *a)* above satisfies $u(x,0) = 1$ and $\frac{\partial u}{\partial t}(x,0) = 3$ for $-L \leq x \leq L$. Write a formula for this solution $u(x,t)$. (Hint: this solution corresponds to the eigenvalue $\lambda = 0$ of the Sturm-Liouville problem for ϕ.)

5. *(25 points)* Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

for $u(x,t)$ on $0 < x < 1$, $t > 0$, with boundary conditions

$$u(0,t) = 0 \quad \text{and} \quad \frac{\partial u}{\partial x}(1,t) + u(1,t) = 0$$

for $t \geq 0$, and initial condition

$$u(x,0) = f(x)$$

on $0 \leq x \leq 1$, where $f(x)$ is a given function.