Math 4163 — Spring 2013
Review for Exam 2

Exam 2 covers sections 2.5.2, 3.2, 3.3, 4.4, 5.3, and 5.8 of the text. The relevant assignments are Assignments 4, 5, 6, and 7. Here is a study guide for the material we covered in these sections.

2.5.2. Laplace's equation for a circular disk. We covered all the material in this section. The method used in this section is the same as was used for all the previous problems we've covered: namely, separation of variables and Fourier series. Also the eigenvalue problem for the function $\phi(\theta)$ involves the same ODE as the eigenvalue problems covered previously. The main new aspect of the problem covered in this section is that the ODE for the function $G(r)$ is a little more complicated than the ODE for the function $G(t)$ we encountered in solving the heat equation. Also, the boundary condition at $r = 0$ is different from the other one's we've seen: it's not a condition on the value of $G(0)$ or $G'(0)$, but rather the condition that $G(r)$ does not go to infinity as r approaches zero.

You should be familiar with the expression of Laplace's equation in polar coordinates, equation (2.5.30). You don't need to memorize it, but you should be used to working with it.

3.2, 3.3. Convergence theorems for Fourier series and Fourier cosine and sine series. You should review these sections in their entirety. Section 3.3 is a little long, not because there's a whole lot of material in it, but because the author goes through a number of examples in detail. A good method for reviewing it might be to skim it first and try to do some of problems 3.3.1 through 3.3.6 at the end of the chapter; then compare your answers to the ones at the end of the book.

4.4. Vibrating string with fixed ends. We covered most of this section, except the last paragraph, which you can skip. The new mathematical feature of the problem discussed in this section is that there are two initial conditions to satisfy, instead of just one as for the heat equation. These are used to find the two sets of constants A_n and B_n in the formula (4.4.11) for the solution of the problem.

It's also a good idea to have at least a rudimentary understanding of the physical situation behind the problem: what u stands for here, what c means, what the different boundary conditions mean physically, what the terms "normal mode" (or "mode of vibration") and "natural frequencies" refer to. Generally, the (circular) frequency of an oscillating motion given by $\sin(\omega t)$ is the constant ω, which has units of radians per second. For the wave equation, the normal models are the separated solutions $\phi_n(x)(A\cos(c\sqrt{\lambda_n}t) + B\sin(c\sqrt{\lambda_n}t)$ corresponding to the eigenfunctions ϕ_n of the Sturm-Liouville problem with eigenvalues λ_n, the natural frequencies are the frequencies $c\sqrt{\lambda_n}$ of these modes.

5.3. Sturm-Liouville eigenvalue problems. We covered everything in this section (including the material about Rayleigh quotients, which I skipped in last year's class, but decided to cover this year). The content of this section boils down to the facts mentioned in the box on page 157. You don't need to memorize them — to appreciate them, I would recommend briefly going over the problems we've done all semester (from the first assignment all the way up through section 5.8), and checking to see how the theorems in the box on page 157 apply to them.

5.8. Boundary conditions of the third kind. We covered this entire section in class. Again, the section seems a little long at first, but it's only because it covers several examples in detail; there isn't a whole lot of new material here that wasn't in previous sections. The main new wrinkles are that (i) we can't find simple formulas for the eigenvalues, but have to solve for them graphically (remember, calculator's aren't allowed on the exam); (ii) there isn't a simple formula for the integrals $\int_0^L \phi_n(x)^2\,dx$ which come up in the formulas for the coefficients of the series for a_n, so we just leave these unevaluated in the solution (see for example the equation for a_n on page 204); and (iii) in some of these problems, negative eigenvalues do exist, unlike the situation in previous chapters where the only eigenvalues were positive or zero.