Math 4163 — Spring 2013
Review for Exam 3

Exam 3 covers portions of sections 7.2, 7.3, 7.4, 7.6, 7.7, and 7.8 of the text. The relevant assignments are Assignments 8 and 9. Here is a study guide for the material we covered in these sections.

7.2. Separation of the time variable. This section just reviews what happens when you separate the time variable from the space variables in the two- or three-dimensional heat or wave equations. You get a first- or second-order ordinary differential equation for a function $h(t)$ of time; and a two- or three-dimensional eigenvalue problem for a function ϕ of the space variables: that is, a problem of the form $\nabla^2\phi = -\lambda\phi$ on some region R in two- or three-dimensional space, where ϕ is supposed to satisfy certain boundary conditions on the boundary of R. (In the lectures I often used Δ in place of ∇^2 to stand for the Laplace operator. A good summary of how Laplace's operator is defined can be found at the Wikipedia article for "Laplace operator".)

7.3. Vibrating rectangular membrane. We covered this whole section. The key results are in equations (7.3.25), (7.3.26), (7.3.27), (7.3.30), and (7.3.32). You should definitely review these, but I wouldn't spend time or effort memorizing them. Remember the idea is to know enough about how these results are derived to be able to figure out how they would change if different boundary conditions, or a different partial differential equation, were given.

7.4. Statements and illustrations of theorems for the eigenvalue problem. This section consists of some useful general facts about the eigenvalue problem $\nabla^2\phi = -\lambda\phi$, listed in the box on page 283, followed by a discussion of how these facts apply to the specific case when the region R being considered is a rectangle, and the boundary conditions are Dirichlet boundary conditions. You should be familiar with these general facts. You will not need to know about the Rayleigh quotient in (7.4.6), but as I mentioned in class, knowing about this could be helpful in keeping you from going wrong by letting you know when to expect and when not to expect zero or negative eigenvalues. There is more about this in section 7.6.

7.6. Rayleigh quotient and Laplace's equation. As mentioned in the paragraph just above, you won't need to know the material in this section for the test, but it is helpful to know it.

7.7. Vibrating circular membrane and Bessel functions. We covered this entire section, but for this exam, there's no need to review subsections 7.7.5 and 7.7.6: they're just there to help you feel a little more comfortable with Bessel functions by describing what they look like near $z = 0$ and $z = \infty$. You do not need to memorize Bessel's equation (7.7.25), or the expression of Laplace's equation in polar coordinates (7.7.9) — I'll provide those on the exam where needed — but you should be used to working with them, and if you've done that you've probably almost memorized them already.

7.8. More on Bessel functions. Like subsections 7.7.4 and 7.7.6, this section does not contain anything that you absolutely need to do the exam, but reviewing some of it, especially subsection 7.8.3 on the zeros of Bessel functions, could be helpful. You can skip the concluding subsection 7.8.4 entirely, though.