Math 4163 — Review for First Exam

In preparing for the exam, you should first review the problems on Assignments 1 and 2, and then look at similar problems from the problem sections at the end of sections 2.3 and 2.4. Also, you should read through sections 2.1, 2.2, 2.3, and 2.4.1 of the text. I've included a guide to reviewing these sections below.

At the exam, I will hand out a sheet you can use during the exam, reproducing the chart on the inside front cover of the text, titled "EIGENVALUE OR BOUNDARY VALUE PROBLEMS for $\frac{d^2\phi}{dx^2} = -\lambda\phi$".

2.1, 2.2. You should read through these short sections just to familiarize yourself with the type of problem we are solving, and some of the terminology we use.

2.3. This is the key section for this exam. It explains how the method of separation of variables works when applied to the initial-boundary-value problem for the heat equation given in equations (2.3.1), (2.3.2), and (2.3.3) on page 35. You should know pretty much everything in this section.

Notice that in class I did things in a slightly different order than in the text. First I explained the material in section 2.3.4 about how to solve eigenvalue problems. Next I explained the material in section 2.3.6 about how to express a function $f(x)$ as a linear combination of eigenfunctions (in this case $\sin(n\pi x/L)$), using the orthogonality of the eigenfunctions. And only then did I explain the method of separation of variables, as given in sections 2.3.2, 2.3.3, and 2.3.5.

Note that the final solution to the boundary-value problem in (2.3.1), (2.3.2), and (2.3.3) is given by the formula for $u(x,t)$ given in equation (2.3.30), together with the formula for the coefficients given in equation (2.3.35).

The example in section 2.3.7 is important, showing what the solution would look like when a specific function $f(x)$ is given as the initial data (in this case, $f(x) \equiv 100$).

2.4. You should read carefully through section 2.4.1. (We'll discuss 2.4.2 in class next week, but the material in 2.4.2 won't be part of the first exam.) In section 2.4.1, the method of separation of variables is again applied to an initial-boundary value problem for the heat equation, but this time the boundary conditions are different, so we wind up with different eigenfunctions and eigenvalues. The problem being solved is given in equations (2.4.1), (2.4.2), (2.4.3), and (2.4.4); and its solution is given by the series for $u(x,t)$ appearing in (2.4.19), together with the formulas for the coefficients appearing in (2.4.23) and (2.4.24).

Actually, the eigenvalues for the problem in section 2.4 turn out to be almost the same as the eigenvalues for the problem in section 2.3; with one important difference: here $\lambda = 0$ is an eigenvalue, while in section 2.3 it was not. You should understand clearly how we determined that this was the case. See the paragraph halfway down page 57, starting with "If $\lambda = 0 \ldots$ ", and compare to the paragraph at the top of page 41, starting with "Eigenvalue ($\lambda = 0$)" (a little bit deceptively, since $\lambda = 0$ is not an eigenvalue here).