Math 4163 — Review for Second Exam

In preparing for the exam, you should first review the problems on Assignments 3, 4, and 5, and then review the contents of sections 2.5, 3.1, 3.2, 3.3, and 4.4. I've included a guide to reviewing these sections below.

At the exam, I will hand out a sheet you can use during the exam, reproducing the chart on the inside front cover of the text, titled "EIGENVALUE OR BOUNDARY VALUE PROBLEMS for $\frac{d^2\phi}{dx^2} = -\lambda\phi$".

2.5. At the beginning of this section, the author points out that the problem given in equations (2.5.1) through (2.5.5) (i.e., Laplace's equation on a rectangle with four inhomogeneous boundary conditions) can be solved by finding the solution of four separate problems, each with three homogeneous boundary conditions and one inhomogeneous boundary condition, and then adding up the solutions of these four problems. (By "homogeneous" boundary condition I mean one where the unknown u, or its derivatives, or some combination of u and its derivatives, is set equal to zero.)

Each of these four problems could be solved the same way, and the process is illustrated by solving one of them, namely the one given in equations (2.5.7) through (2.5.11). You should review this, and should be able to solve similar problems, such as the ones on Assignment 3.

We also discussed section 2.5.2, Laplace's equation inside a circular disk, in class. Besides going completely through the solution of the problem in this section, we covered several similar problems on Assignment 4.

You can skip sections 2.5.3 and 2.5.4. Although, if you're interested, you might take a look at least at the paragraph on page 80 about well-posedness and uniqueness, which are important topics in the subject of partial differential equations.

3.1, 3.2, 3.3. You should read through section 3.1 for background in understanding the other two sections.

You should know the definition of the *Fourier series* of a function $f(x)$ defined for $-L \leq x \leq L$. This is given by the series in (3.2.1), with coefficients as in (3.2.2). You know where these formulas for the coefficients come from already, because we talked about how to derive them earlier, in section 2.4.2.

You should read the theorem in the box on page 89. You don't need to know the details of this theorem for the exam, but it contains the main important idea in this chapter, which is that the Fourier series of a function actually converges to the function at most points, provided the function is reasonably well-behaved.

You should read the example on pages 90 and 91 carefully. I might ask you to work out a similar example, like those in problems 3.2.1 or 3.2.2 on page 92.

If a function $f(x)$ is defined for $0 \leq x \leq L$, we can define its *Fourier sine series* by (3.3.5), with coefficients as in (3.3.6). Notice the difference between the coefficients in (3.3.6) and those in (3.2.2). For one thing, the integral on (3.3.6) is only between $x = 0$ and $x = L$, as it had better be, because the function we're considering is only defined between $x = 0$ and $x = L$.

The author makes the point that the Fourier sine series of a function f which is defined on $[0, L]$ is actually the same as the Fourier series of the *odd extension* of f to $[-L, L]$. This can be a bit confusing at first, but is important; you should think it over until you feel you understand it clearly. Read carefully the box at the top of page 95, the example on pages 95 to 97, and the "further example" on page 100. The example of the Fourier sine series of the cosine function, which starts at the bottom of page 100 and goes to page 102, is also worth reading. You can skip the material about the Gibbs phenomenon on pages 97 to 100 if you like (though those of you who like numerical computation might find it interesting).

For functions $f(x)$ defined for $0 \leq x \leq L$, the *Fourier cosine series* is defined by (3.3.18), with coefficients as in (3.3.19) and (3.3.20). Again, the author points out that the Fourier cosine series of f is actually the Fourier series of the *even extension* of f to $[-L, L]$. You should read through sections 3.3.2 and 3.3.3 carefully.

You can skip section 3.3.5 if you like.

4.4. In class, we mostly skipped the material in sections 4.1 to 4.3 which explains the physical meaning of the initial-boundary-value problems for the wave equation that we consider later. However, you might like to refer back to these two sections if you want to gain some physical intuition about the problems we're solving in this chapter. At least, you should know that the wave equation (4.4.1) with Dirichlet boundary conditions (4.4.2) describes the motion of an elastic string of length L whose ends, at $x = 0$ and $x = L$, are tacked down to the x-axis (and so do not move). Also, the initial conditions (4.4.3) describe the initial vertical displacement $u(x, 0)$ and the initial vertical velocity $u_t(x, 0)$ of the string.

In class, we went through the discussion in section 4.4 in detail in class. In fact, we went further and talked about D'Alembert's solution of the wave equation. If you missed class that day, you can find a summary of D'Alembert's solution on pages 550 and 551 of the text — it's quite easy to understand. You don't need to know about D'Alembert's solution for this exam. But you should be able to solve problems for the wave equation and related equations, like problems 2 and 3 on Assignment 5, for example.