Math 4163 — Review for Third Exam

The third exam is over sections 5.8, 7.3 and 7.7 of the text.
I've included a guide to reviewing these sections below.

5.8. Boundary Conditions of the Third Kind. The material in this section is not much different from what we had studied before. As before, when solving the PDE problem by separation of variables, we encounter the equation $\phi''(x) + \lambda\phi(x) = 0$ with two boundary conditions. The main differences from the problems we studied before are that (1) we get equations for the eigenvalues that can't be solved explicitly, like $\tan\sqrt{\lambda} = -2\sqrt{\lambda}$; and (2) sometimes the problems have negative eigenvalues.

However, the point of the section is that despite these differences, the process of solving the PDE still goes overall the same way it did before: we find a complete set of eigenfunctions which are orthogonal to each other, and when we express the desired solution as a linear combination of separated solutions, we can use these facts to find the coefficients in the linear combination.

This section analyzes a few problems in detail and writes up the results in a table. You should read through the section, but there would be no point in trying to memorize the results. It's understanding the process used to obtain them that's important.

One other new thing in this section was the Rayleigh quotient. The formula for the Rayleigh quotient, which I gave in class, is to be found in the text on the box numbered (5.6.3) in section 5.6. (Don't worry about the p, q, and σ in this formula; here we're only considering the case when they're all equal to 1.) You don't need to memorize this formula; I'll give it to you if I ask a question about it. But you should understand how it can be used to show that an eigenvalue problem has no negative eigenvalues, as on page 200, or as I did in class.

7.3. Vibrating Rectangular Membrane. What's new in the problems discussed in this section is that when you apply the method of separation of variables, you have to separate variables twice. Starting from (7.3.1), we first separate out the t variable from the x and y variables to get (7.3.7) and (7.3.8). Then we solve (7.3.8) by separating the x from the y variable, as in (7.3.13) and (7.3.14). This gives us (7.3.15) and (7.3.16). (The process is summarized again a little more neatly on pages 281 to 282.) I might ask a question on the exam where you have to do something like that.

You should review the entire section, and at least think a bit about how to do the problems at the end of the section.

7.7. Vibrating Circular Membrane and Bessel Functions. You should review sections 7.7.1, 7.7.2, 7.7.3, 7.7.4, 7.7.7, 7.7.8, and 7.7.9. You can skip sections 7.7.5 and 7.7.6.

You do not need to memorize the Laplacian in polar coordinates ($\Delta\phi = \phi_{rr} + \frac{1}{r}\phi_r + \frac{1}{r^2}\phi_{\theta\theta}$) or Bessel's equation (7.7.25); if I ask questions about them I'll provide the equation. But you should be pretty familiar with the equations by now, having worked several problems involving them.