**Introduction to Analysis**
**Exam 1**

**1.** *(5 points)* Give the definition of supremum of a set.

**2.** *(5 points)* State the completeness property of real numbers.

**3.** *(5 points)* Give the definition of limit of a sequence.

**4.** *(20 points)* Let $S$ be a nonempty set of real numbers, and let $T = \{|x| : x \in S\}$. Suppose $S$ and $T$ are bounded.

**a.** Show that $\sup S \leq \sup T$.
**b.** Give an example to show that $\sup S = \sup T$ may be false.

**5.** *(15 points)* Find $\lim \left( \dfrac{3n^2 + n}{5n^2 - 6} \right)$. You may use any result from class, but you should give a complete proof.

**6.** *(30 points)*
**a.** Use induction to prove that $4^n \geq n^2$ for all $n \in \mathbf{N}$.

**b.** Use part **a.** to prove that $\lim \left( \dfrac{n}{4^n} \right) = 0$.

**7.** *(20 points)* Let $(x_n)$ and $(y_n)$ be sequences. Suppose that $(x_n)$ converges to 0, and suppose that $|y_n| < 2$ for all $n \in \mathbf{N}$. Prove that $(x_n y_n)$ converges to 0.

(Note: for this problem you cannot assume that $(y_n)$ converges, so you cannot use the theorem from the text about products of convergent sequences.)