Review for First Exam

The first exam will cover Sections 2.1, 2.2, 2.3, 2.4, 3.1 and 3.2 of the text. The problems should be similar to those on the first ten homework assignments. I might also ask you for one or more of the following:

- definition of upper bound and of supremum of a set
- statement of the completeness property of real numbers
- definition of limit of a sequence.

Here is a more detailed list of topics from the text to review for the exam.

2.1. The algebraic and order properties of R. The basic algebraic and order properties of **R** are already familiar to you from more elementary math courses. The main point of this section is that everything we do in this course (and in any more advanced analysis course as well) can be stated as a sequence of definitions and theorems, all of whose proofs boil down in the end to logical deductions from a small collection of axioms — the algebraic properties given in 2.1.1 and the order properties given in 2.1.5 (together with the completeness property described later in 2.3.6). You do not need to know what these specific algebraic and order properties are, but you should have a feel by now for what kind of statements you can use in proofs with confidence that they are ultimately justifiable by these axioms.

For this test, you won't actually need to know anything about rational vs. irrational numbers (see page 25 and 26), but I recommend glancing through these paragraphs anyway — we will come back to this topic soon enough, as it's important later in the course.

I will expect you to be familiar with Bernoulli's inequality (page 30), which we've already used a couple of times in the exercises. Recall that Bernoulli's inequality was proved using mathematical induction, and that we also have used induction several times for the homework exercises already. You should be able to do simple proofs by induction like the ones assigned from section 1.2.

2.2. Absolute value and the real line. I find that a good predictor of a student's success in analysis is how comfortable they are with manipulating absolute values and inequalities correctly. To do this it helps to be able to easily visualize what a statement about absolute values means geometrically. So I recommend trying to get this section down pat.

2.3. The completeness property of R. We covered this section in its entirety; you should be quite comfortable with its contests. In particular, as mentioned above, you should start by making sure you have the definitions of "upper bound" and "supremum" and the statement of the Completeness Property (2.3.6) memorized.

2.4. Applications of the supremum property. In reviewing this section, you can skip the subsections titled "Functions", "The existence of $\sqrt{2}$" and "Density of rational numbers in **R**".

3.1. Sequences and their limits. We covered this section in its entirety. Understanding this section is crucial to making sense of the rest of the course.

3.2. Limit theorems. In this section, the definition of limit used in the preceding section is used to prove basic theorems about limits. The more you learn from this section before the exam, the better; but at least you should know Theorems 3.2.1 through 3.2.7. I also strongly recommend reading Examples 3.2.8(a,b,c,d,e,f).