Introduction to Analysis
Exam 1

1. *(25 points)*
a. Show that if $|x-3|<1$ then $|x-6|>2$.

b. Show that if $|x-3|<1$ then $\left|\dfrac{x^2+1}{x-6}\right|<\dfrac{17}{2}$.

2. *(15 points)* Suppose S and T are nonempty bounded sets and $\sup S > \inf T$. Show that there exist numbers $x \in S$ and $y \in T$ such that $x > y$.

3. *(20 points)* Define $I_n = (0, 1+1/n)$ for each $n \in \mathbf{N}$. Show that if x is in I_n for every n, then $x \leq 1$.

4. *(25 points)* Show that the following sequences converge, and find the limits. You may use any result from class, but you should give a complete proof.

a. $x_n = \left(\dfrac{n^2}{n+1}\right) - \left(\dfrac{n^3-3n^2-2}{n^2+n}\right)$

b. $y_n = \dfrac{(-1)^n}{\sqrt{n}}$

5. *(15 points)* Show that the sequence $\left(1+\dfrac{2}{n}\right)^{n^2}$ does not converge. (Hint: use Bernoulli's inequality.)