

**Introduction to Analysis**  
**Exam 2**

You may use any result from class, but when you do so please try to be clear as to which result you are using.

1. (25 points) Let  $x_1 := 2$  and  $x_n := \sqrt{x_n - 1} + 3$  for  $n \in \mathbf{N}$ .

a. Show that  $x_n$  is bounded and monotone.

b. Find the limit of  $(x_n)$ .

2. (15 points) Suppose  $(x_n)$  is a sequence such that  $\lim(x_n) = 0$ . Show that the sequence  $(1/x_n)$  does not converge.

3. (15 points) Suppose  $(x_n)$  has one subsequence in which all the terms are greater than or equal to 2, and another subsequence in which all the terms are less than or equal to 1. Prove that  $(x_n)$  does not converge.

4. (10 points) Find  $\lim \left(1 + \frac{1}{3n}\right)^n$ .

5. (20 points) Find whether the following series converge. Prove your answer.

a.  $\sum_{n=1}^{\infty} \frac{2^n}{n^n}$

b.  $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$

6. (15 points) Suppose  $f$  is a function defined on  $\mathbf{R}$ , and let  $g$  denote the function defined for  $x \in \mathbf{R}$  by  $g(x) := f(x - 2)$ . If  $\lim_{x \rightarrow 0} f(x)$  exists, prove that  $\lim_{x \rightarrow 2} g(x) = \lim_{x \rightarrow 0} f(x)$ .