Review for Third Exam

The third exam (Friday, July 6) will cover portions of Sections 4.1, 4.2, 5.1, 5.2, 5.3, 6.1, and 6.2 of the text. We've had homework problems from these sections on assignments 11 through 16.

Here's a summary of what portions of the text will and will not be covered on the exam. You should also go through the lecture notes and compare them to what's in the text.

4.1. Limits of functions. 4.2. Limit theorems. We've covered everything in these two sections by now, except that perhaps you aren't familiar with the inequalities for trigonometric functions that are used in Examples 4.2(b) through 4.2(f). These inequalities can be easily proved using the Mean Value Theorem. I'll go over a couple of them on Thursday, the day before the text.

5.1. Continuous functions. 5.2 Combinations of continuous functions. We've also covered everything in these two sections, with the exception of the subsection titled "Composition of continuous functions" on pages 132 and 133. However, I think it would be helpful for you to know what's in this subsection — reading it doesn't require you to learn any new concepts, and if you know what's in this section it will make working with continuous functions a lot easier. For example, once you know Theorem 5.2.7 it's easy to see that the function $f(x) = \sqrt{x^2 + 1}$ is continuous on $\mathbf{R}$, whereas it would take some time and effort to prove this without using Theorem 5.2.7.

So I'll briefly discuss composition of continuous functions on Thursday as well.

5.3. Continuous functions on intervals. We covered only pages 134 to 136 from this section: the material leading up to and including the "Maximum-Minimum Theorem", or, as we called it in class, the "Extreme Value Theorem". There were no homework problems assigned from this section, but we did use the Extreme Value Theorem in class later on to prove Rolle's theorem, and it's possible I might ask for a similar simple application of the Extreme Value Theorem. You will be okay for the exam if you know what the Extreme Value Theorem says.

Unfortunately, we didn't have time in class to cover the other major topic in this section, the Intermediate Value Theorem. It won't appear on the exam, but there are several other math courses or topics you might study later (including numerical analysis and topology) for which it would be quite useful to know the Intermediate Value Theorem. So this is one you might want to come back to later on, after the class is over.

6.1. The derivative. In this section we only covered the material from page 162 through the first half of page 165: up to, but not including, the subsection titled "The Chain Rule". In fact, we did not actually cover all of Theorem 6.1.3 either, since we didn't prove the Quotient Rule in class.

The Chain Rule and Quotient Rule are rules you already know from calculus class, though you may not have seen their proofs before. If we have time on Thursday, we'll

briefly discuss the proof of the Quotient Rule. It's not a proof you'll specifically need to know for the exam, but the idea behind the proof is useful to know.

The proof given in the text for the Chain Rule, on the other hand, is not very edifying, or at least doesn't seem to include ideas that are useful in other contexts, so we can safely skip it in this class. (There are other proofs of the Chain Rule that do introduce widely useful ideas, but they require introducing too mathematical machinery for a first analysis course. Pretty much the same comments apply to the theorem on derivatives of inverse functions, which we have also skipped in this course.)

6.2. The Mean Value Theorem. So far we've covered from the beginning of the section through the Mean Value Theorem on page 174, and we've done one of the examples on page 177. (A couple of similar examples were on the last homework assignment.) On Thursday I want to do at least a couple more short examples, including Theorem 6.2.5 or 6.2.7. You will not need to know the remainder of the section for this exam. That is, you can skip Theorem 6.2.8, Examples 6.2.9, and the subsection titled "The Intermediate Value Property of Derivatives".