Math 4443/5443
Quiz (take-home)

Instructions: Prove the following statements. Please indicate which theorems you are using in your proof.

1. Show that the series

$$P(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

and

$$Q(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

converge pointwise on $\mathbf{R}$ and uniformly on $[-T, T]$, for every $T > 0$.

2. Show that $P(x)$ and $Q(x)$ are differentiable on $\mathbf{R}$, and that for all $x \in \mathbf{R}$,

$$\begin{aligned} P'(x) &= Q(x), \\ Q'(x) &= P(x), \\ P''(x) &= P(x), \\ Q''(x) &= Q(x). \end{aligned}$$

3. Show that $(P(x))^2 - (Q(x))^2 = 1$ for all $x \in \mathbf{R}$.