Math 5423
What Exam 2 Covers

The second exam will cover all of chapter 3 and sections 4.1 through 4.5 of the text.

As was true for the first two chapters, there were only minor differences between the presentation in the book and the lectures, apart from the fact that in class I did several topics in a different order than they occurred in the lectures. As an example of that, in class I first proved the general theorem that the uniform limit of holomorphic functions is holomorphic, and then used that general theorem to prove that the sum of a convergent power series is holomorphic. In the text, the authors first prove that the sum of a convergent power series is holomorphic by a special argument, in Lemma 3.2.10; and then give the more general theorem later, in Theorem 3.5.1.

There was one important theorem in Chapter 3 of the text which I haven't covered yet: Morera's Theorem (Theorem 3.1.4). The reason I say this theorem is important is that in most complex analysis texts it is used to prove Goursat's theorem, which is the theorem that a complex differentiable function is holomorphic, even if we don't assume the function is C^1. Since we haven't proved or used Goursat's theorem in class yet (it is proved in the text on pages 493–495), we haven't needed Morera's theorem yet. But I think I will take a bit of time in class, after we get back from Thanksgiving, to prove Morera's theorem and Goursat's theorem — the proofs are not long, and they are edifying.

Another difference between the lectures and the text is that in class we've gone a bit more carefully through the definition and properties of the complex exponential, trigonometric, and logarithm functions than the text has — especially the logarithm, which is hardly discussed at all in the text. If you would like to see a definition of the logarithm written down somewhere besides in my notes, you can find a pretty good discussion in the Wikipedia article on "Complex logarithm". The main omission from the Wikipedia article is the proof that a branch of the logarithm function is holomorphic where defined: I proved this in class by first proving the general result that the inverse of a holomorphic function f exists and is holomorphic in some neighborhood of any point z_0 where $f'(z_0) \neq 0$. (The Wikipedia article refers to this result as the "complex version of the inverse function theorem", but does not provide a proof.)

The class discussion of meromorphic functions and residues pretty much covered everything that is contained in sections 4.1 through 4.5 of the text. One minor difference is that I first proved the residue theorem for the special case in which the curve γ is a simple, positively oriented, closed curve; and only later gave the more general version with indexes (Theorem 4.5.3).