Complex Analysis II
Assignment 6

1. Show that if $|z| < 1$ then

$$(1 + z)(1 + z^2)(1 + z^4)(1 + z^8) \cdots = \frac{1}{1 - z} \ .$$

2. Suppose that $\{a_n\}$ is a sequence of distinct complex numbers, and $\{A_n\}$ is an arbitrary sequence of complex numbers. Show that there exists an entire function $f(z)$ such that for all $n \in \mathbf{N}$, $f(a_n) = A_n$.
(Hint: Let $g(z)$ be a function with simple zeroes at the a_n. Show that

$$\sum_{n=1}^{\infty} g(z) \frac{e^{\gamma_n(z - a_n)}}{z - a_n} \frac{A_n}{g'(a_n)}$$

converges for some choice of the numbers γ_n.)