Math 5463 — Review for Exam 1

Exam 1 covers chapter 7 of Royden and Fitzpatrick's test.

7.1. We've covered everything in this section.

7.2. We covered everything in this section up through the Cauchy-Schwarz inequality on page 142. However, I gave different proofs of some of the inequalities.

You can skip Corollary 2, if you like. Corollary 3 and the two examples at the end of the section are similar to some things we did in class, and are worth looking at. However, there is a mistake in the last example. The set E should be $(0, \infty)$, not $(1, \infty)$, and the function f should be defined by

$$f(x) = \frac{x^{-1/2}}{1 + |\ln x|} \quad \text{for } x > 0.$$

(Note: for a list of some of the other mistakes in Royden and Fitzpatrick, go to http://www2.math.umd.edu/∼ pmf/RAE.pdf. You can find this page by Googling "Royden Fitzpatrick errata".)

7.3. This section contains three results: the Riesz-Fischer theorem, Theorem 7 on page 148, and Theorem 8 on page 149. I proved the Riesz-Fischer theorem in class, with a somewhat different proof. I didn't cover Theorems 7 or 8 in class, and they won't be necessary for the exam, but they could be useful. Several students used Theorem 7, for example, to do problem 26 on page 150 — though it is just about as easy to do this problem without Theorem 7.

7.4. We covered this entire section in class. We also gave a nice illustration in class of a typical use of Theorem 12, by using it to prove that if $f \in L^p$, $1 \le p < \infty$, then

$$\lim_{h \to 0} \|f(x+h) - f(x)\|_p = \lim_{h \to 0} \left(\int |f(x+h) - f(x)|^p \, dx \right)^{1/p} = 0.$$