

1. Problem 60, chapter 18 (on page 387) of Royden and Fitzpatrick.

2. (This is problem 24 from chapter 10 of Wheeden and Zygmund.) Let (X, Σ, μ) be a σ -finite measure space, and let f be Σ -measurable and integrable over X . Let Σ_0 be a σ -algebra satisfying $\Sigma_0 \subseteq \Sigma$. Of course, f may not be Σ_0 -measurable. Show that there is a unique function f_0 which is Σ_0 -measurable such that $\int fg \, d\mu = \int f_0 g \, d\mu$ for every Σ_0 -measurable g for which the integrals are finite. (The function f_0 is called the *conditional expectation* of f with respect to Σ_0 , denoted $f_0 = E(f|\Sigma_0)$.) *Hint:* Apply the Radon-Nikodym theorem to the set function $\phi(A) = \int_A f \, d\mu$, $A \in \Sigma_0$.