Math 5463 — Review for final

The material we've covered in class corresponds, more or less, to the following sections of Royden and Fitzpatrick:

7.1, 7.2, 7.3, 7.4, 13.1, 13.2, 16.1, 16.3, 17.1, 17.2, 17.3, 17.4, 17.5, 18.1, 18.2, 18.3, 18.4, 19.2, 20.1;

or the following sections of Wheeden and Zygmund:

6.1, 8.5, 8.6, 8.7, 10.1, 10.2, 10.3, 10.4, 11.1, 11.5.

On the review sheet for the first exam, I gave a bit of detail on what parts of chapter 7 of Royden and Fitzpatrick we've covered in class. Here are some quick notes on the remaining material.

Sections 13.1 and 13.2 just cover some basic terminology concerning normed spaces; many of you have seen this material before in other classes.

Sections 16.1 and 16.3 cover most of the material that we did in class on Hilbert spaces. I followed Wheeden and Zygmund for this material, but the treatment in Royden and Fitzpatrick should provide a useful review.

We have covered in class just about all the material in chapter 17, except not in the same order presented there. The main difference between what's in the text and what I did in class is that the text considers premeasures in slightly more generality than I did in class, and therefore gives different definitions. Following Wheeden and Zygmund, I only talked about premeasures on algebras and semi-algebras, not on general collections of sets. If I ask anything on the exam about premeasures, or algebras, or semi-algebras, or countable monotonicity, I'll supply a definition with the question so you'll know what the term is supposed to mean. If you're getting confused about these basic definitions, a good idea is to try the problems from pages 352 and 357 that I suggested. Work on them for a while before looking at the answers I posted online! This is the best way to appreciate how all the definitions fit together.

We also covered pretty much all the material in sections 18.1 through 18.4. My definition of the integral of a measurable function (again, following that in Wheeden and Zygmund) looks different from that given in Royden and Fitzpatrick, but the two definitions are actually equivalent. If you want to use one on the test while doing a problem, it doesn't matter which one you use.

Section 19.2 contains a statement of the "L^p duality" theorem — the theorem that the space of bounded linear functionals on L^p is $L^{p'}$ — and a proof of the theorem as a consequence of the Radon-Nikodym theorem. I followed Wheeden and Zygmund for this material in class, and though I haven't checked all the details of the proof in Royden and Fitzpatrick, from what I've seen it looks exactly the same.

Section 20.1 contains a proof of Fubini's and Tonelli's theorems. We (almost) completed a proof of Fubini's theorem in class, and didn't get to Tonelli's theorem. I won't ask about these on the final. They are standard topics on the analysis qualifying exam, though. Despite the fancy names, in practice they really just say that you can evaluate a multiple integral by iterated integration (the way you learn to do it in calculus), as long as the function you are integrating is integrable in the product space. There is a nice section on applications of this theorem, with exercises, in chapter 6 of Wheeden and Zygmund's text.