***7.7.1.** Solve as simply as p

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

with $u(a,\theta,t) = 0, u(r,\theta,0) = 0$ and $\frac{\partial u}{\partial t}(r,\theta,0) = \alpha(r)\sin 3\theta$.

7.7.2. Solve as simply as possible:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u \text{ subject to } \frac{\partial u}{\partial r}(a,\theta,t) = 0$$

with initial conditions

(a) $u(r,\theta,0) = 0,$ $\quad \frac{\partial u}{\partial t}(r,\theta,0) = \beta(r)\cos 5\theta$

(b) $u(r,\theta,0) = 0,$ $\quad \frac{\partial u}{\partial t}(r,\theta,0) = \beta(r)$

(c) $u(r,\theta,0) = \alpha(r,\theta),$ $\quad \frac{\partial u}{\partial t}(r,\theta,0) = 0$

***(d)** $u(r,\theta,0) = 0,$ $\quad \frac{\partial u}{\partial t}(r,\theta,0) = \beta(r,\theta)$

7.7.3. Consider a vibrating quarter-circular membrane, $0 < r < a, 0 < \theta < \pi/2$, with $u =$ on the entire boundary. [*Hint*: You may assume without derivation that $\lambda > 0$ an that product solutions

$$u(r,\theta,t) = \phi(r,\theta)h(t) = f(r)g(\theta)h(t)$$

satisfy

$$\nabla^2\phi + \lambda\phi = 0$$

$$\cancel{\frac{dh}{dt} = -\lambda k h} \leftarrow \boxed{\frac{d^2h}{dt^2} = -c^2\lambda h}$$

$$\frac{d^2 g}{d\theta^2} = -\mu g$$

$$r\frac{d}{dr}\left(r\frac{df}{dr}\right) + (\lambda r^2 - \mu)f = 0.]$$

***(a)** Determine an expression for the frequencies of vibration.

(b) Solve the initial value problem if

$$u(r,\theta,0) = g(r,\theta), \qquad \frac{\partial u}{\partial t}(r,\theta,0) = 0.$$

(c) Solve the wave equation inside a quarter-circle, subject to the conditions

boundary conditions: →

$$\frac{\partial u}{\partial r}(a,\theta,t) = 0, \qquad u(r,0,t) = 0$$

$$u\left(r,\frac{\pi}{2},t\right) = 0, \qquad u(r,\theta,0) = 0$$

initial conditions →

$$\frac{\partial u}{\partial t}(r,\theta,0) = \beta(r,\theta)$$