Answers to even-numbered problems in Chapter 3

Remember, the answers given below are just intended for you to use to check your work on the assignments. If you were to submit answers to these questions on a homework assignment, quiz, or exam, you would be expected to show the work you did to obtain the answers as well!

Asst. 4

3.1 #2: $a = -2$, $b = -2$, $c = -5$.

3.1 #12: (a)$\mathbf{u}+\mathbf{v} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$, $\mathbf{u}-\mathbf{v} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, $2\mathbf{u} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$, $3\mathbf{u}-2\mathbf{v} = \begin{bmatrix} 10 \\ -1 \end{bmatrix}$; (b) $\mathbf{u}+\mathbf{v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $\mathbf{u}-\mathbf{v} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, $2\mathbf{u} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$, $3\mathbf{u}-2\mathbf{v} = \begin{bmatrix} -6 \\ 5 \end{bmatrix}$; (c) $\mathbf{u}+\mathbf{v} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$, $\mathbf{u}-\mathbf{v} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, $2\mathbf{u} = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$, $3\mathbf{u}-2\mathbf{v} = \begin{bmatrix} 0 \\ 14 \end{bmatrix}$.

3.3 #6: (a) the set is a subspace (b) the set is not a subspace (c) the set is a subspace (d) the set is not a subspace.

3.3 #14: (a) the set is not a subspace (b) the set is a subspace (c) the set is a subspace.

Asst. 5

3.5 #2: (a) the set is not a basis (b) the set is not a basis (c) the set is a basis (d) the set is not a basis.

3.5 #26: (a) 2 (b) 3 (c) 3 (d) 3

Asst. 6

3.6 #2: (a) $x_1 = 2t - s$, $x_2 = s$, $x_3 = t$, where s and t are any real numbers. Other parameterizations are also possible. (b) $\mathbf{x} = s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$. Again, other answers are possible here.

3.6 #8: The only solution is $x_1 = 0$, $x_2 = 0$, $x_3 = 0$. So the solution space contains only one vector, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; i.e., it is the trivial vector space. It has no basis, and its dimension is zero (see Definition 3.11). This problem didn't illustrate the process of finding the basis and dimension of a solution space very well. I should have assigned a problem like #7 in this section instead.

3.7 #2: $[\mathbf{v}]_S = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

3.7 #22: I did this one in class. $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ where

$$\mathbf{w}_1 = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}.$$

Asst. 7

3.8 #6: I did this one in class.

(a) $\{[1 \quad 2 \quad -1 \quad 3], [0 \quad 1 \quad -5 \quad 9], [0 \quad 0 \quad 1 \quad -\frac{8}{7}]\}$. Other answers are possible.

(b) $\{[1 \quad 2 \quad -1 \quad 3], [3 \quad 5 \quad 2 \quad 0], [0 \quad 1 \quad 2 \quad 1]\}$. Other answers are possible.

3.8 #12: (a) row rank = column rank = 3; (b) row rank = column rank = 2; (c) row rank = column rank = 2.