Math 4513
Assignment 4

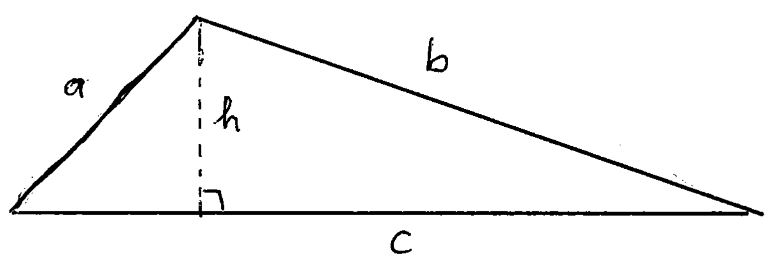
17. In class we showed that if a triangle has sides of length a, b, and c, and if h is the altitude on base c (i.e., h is the length of the perpendicular from side c to the vertex opposite c), then we can find numbers t and u such that

$$a = \frac{h}{2}\left(\frac{1+t^2}{t}\right),$$
$$b = \frac{h}{2}\left(\frac{1+u^2}{u}\right),$$
$$c = \frac{h}{2}\left(\frac{1-t^2}{t} + \frac{1-u^2}{u}\right).$$

Use these formulas, together with the fact that the area A of the triangle is given by $A = \frac{1}{2}ch$, to prove that the area is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \frac{1}{2}(a+b+c)$. This is called *Hero's formula* for the area of a triangle.



18. Suppose d is a rational number. Show that a point $P(x, y)$ on the hyperbola $x^2 - dy^2 = 1$ has rational coordinates x and y if and only if there exists a rational number t such that

$$x = \frac{1+dt^2}{1-dt^2}, \quad y = \frac{2t}{1-dt^2}.$$

19. Show that there are no points $P(x, y)$ on the hyperbola $x^2 - y^2 = 1$ with integer coordinates x and y except for the points $(1, 0)$ and $(-1, 0)$. (Hint: $x^2 - y^2 = (x-y)(x+y)$.)