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Math Capstone

Presentation: The 196 Palindrome

Palindromic Numbers:

Just to make sure everyone is on the same page, a palindromic number is a number that reads the same forwards and backwards.

$a_1a_2a_3\ldots a_3a_2a_1$
e.g. 12321

Reverse Then Add Sum Process

This is the key process in finding palindromes from numbers and the basis behind the 196 phenomenon. To follow the process you add the reverse of a number to itself. Palindromes can come quickly or they may take time. The number of iterations it takes to find a palindrome is (as far as I can tell) random.

For example:

56 becomes a palindrome in only one iteration,

$$\begin{array}{r} 56 \\ \underline{+65} \\ 121 \end{array}$$

But 57 becomes a palindrome in two iterations.

$$\begin{array}{r} 57 \\ \underline{+75} \\ 132 \\ \underline{+231} \\ 363 \end{array}$$

And 59 takes three iterations to be become a palindrome.

$$\begin{array}{r} 59 \\ \underline{+95} \\ 154 \\ \underline{+451} \\ 605 \\ \underline{+506} \\ 1111 \end{array}$$

196 has become a phenomenon because over 400 million iterations have been computed for 196 and it still has not produced a palindrome. It has been termed a Lychrel number. More correctly, it has been termed a "candidate Lychrel number" because it has not been proven that it cannot produce a palindrome.

An interesting side note, Wade VanLandingham is the man who coined the term Lychrel, and he came up with the name after rearranging the letters of his girlfriend's name Cheryl.

There are many complexities to candidate Lychrel numbers.

If you notice, when you put 196 through the Reverse Then Add Process, it is added to 691. Similarly, if you were to take 691 and add it to 196, you would get the same number.

$$\begin{array}{r} 196 \\ +691 \\ \hline 887 \end{array} \qquad \begin{array}{r} 691 \\ +196 \\ \hline 887 \end{array}$$

Also, if you take 295 and put it through the Reverse-Add Sum Process you come up with the same number as 196 after one iteration.

$$\begin{array}{r} 295 \\ +592 \\ \hline 887 \end{array}$$

196, 295, 394, 493, 592, and 691 are considered a "thread" because they all produce the same number (887) after one iteration and then from there it is the exact same sequence of numbers in the Reverse Then Add Process.

196 is considered the "seed" number since it is the lowest number of the thread that does not produce a palindrome. The rest of the numbers are called "kin."

196 – seed
295, 394, 493, 592, 691 - kin
Altogether these numbers produce a thread.

196, 879, 1997 are the only three seeds that exist under 5,000. Most other numbers produce a palindrome within 5 or 6 iterations, and almost all produce a palindrome in 24 iterations.

Conjectures (Erin):

I did many computations of different numbers to try to produce patterns or find something interesting to share. It is very difficult because it all depends up on the order of the numbers and the numbers.

If you will notice, the most important part when using the Reverse Then Add Process is the first and last digit in the number. When those two are added they create a carry, which throws off the entire sum. So, any number greater than 4 will produce a carry.

218
+812
1030
+0301
1331

I decided to share with you the iterations I did with three digits. In these three digit numbers I placed a 1 in the middle. With a 1 in the middle, no matter what the first and last digit are, when the 1 is carried from the sum of the last two digits, you will get a 3 in the middle no matter what. I did many iterations of three digit numbers with only numbers greater than five in the first and last digits and a 1 in the middle. I found many kin numbers within my iterations. I also found that they all produced palindromes within two iterations with the exception of 918, which took four iterations. 719 and 918 did not have kin numbers within my constraints. This is because to produce a 6 or a 7 in the last digit after the first sum, you must have (for 6) 8+8, or 9+7 (for 7) 9+8. 818 would have been a kin number for 719, but it is already a palindrome. It is interesting though that 918 needed four iterations to get to a palindrome whereas all of the others only took two.

617	518	519	618	718	619
+716	+815	+915	+816	+817	+916
1333	1333	1434	1434	1535	1535
+3331	+3331	+4341	+4341	+5351	+5351
4664	4664	5775	5775	6886	6886

516	417	719	918
+615	+714	+917	+819
1131	1131	1636	1737
+1311	+1311	+6361	+7371
2442	2442	7997	9108
			+8019
			17127
			+72171
			89198

I also tried taking the same three digit numbers; with two digits over 5 and a 1, and rearranging them to see if the placement of the 1 affects the number of iterations. I could not find a pattern at all, but here are some examples to share:

178	718	147	417	159	519
+871	+817	+741	+714	+951	+915
1049	1535	888	1131	1110	1434
+9401	+5351		+1311	+0111	+4341
10450	6886		2442	1111	5775
+05401					
15851					

There is no proof in base 10 as to why the candidate Lychrel numbers do not produce palindromes, but there is a proof in base 2.

Proof (Matt):

Empirical evidence suggests that 196 is the smallest number for which there is no palindrome, but there have been no successful attempts at proving this. This may be partially attributed to the large number of pairs that result in the number carries that ruin the palindrome process. As such mathematicians have devised ways to prove that certain numbers in other bases never become palindromes.

Let's consider the number 10110 (22), which is the smallest Lychrel number in base 2. If we initiate the algorithm, we get the following process:

10110
01101
100011
110001
1010100
0010101
1101001
1001011
10110100

The last number of the sequence to the left is of the form:

10[n*1]01[n*0] for n >1

where [n*x] denotes the n repetitions of the number x

From here we can prove inductively that this number is in fact the beginning of a repeating cycle.

Proof:

Step 1: If we iterate the n = 2 case we get the following:

10110100
00101101
11100001
10000111
101101000
000101101
110010101
101010011
1011101000

If we were to put 2 + 1 = 3 into the formula, we would get the number:

10[3*1]01[3*0] = **1011101000**

Step 2: For n >1 we have:

10[n*1]01[n*0] → 10[n-2*1]1101[n-2*0]00 → 11[n-2*0]1000[n-2*1]01
[n*0]10[n*1]01 00[n-2*0]1011[n-2*1]01

11[n-2*0]1000[n-2*1]01 → 101[n-2*1]1010[n-2*0]00 → 10[n*1]01[n+1*0]
10[n-2*1]0001[n-2*0]11

10[n*1]01[n+1*0] → 10[n-1*1]101[n-1*0]00 → 11[n-1*0]010[n-1*1]01
[n+1*0]10[n*1]01 00[n-1*0]101[n-1*1]01

11[n-1*0]010[n-1*1]01 → **10[n+1*1]01[n+1*0]**
10[n-1*1]010[n-1*0]00

Since there are no palindromes during this cycle, we can deduce inductively that any numbers that lead to a number of the form 10[n*1]01[n*0] will never lead to a palindrome.

Similar proofs of bases that are powers of two exist, which leads to the following formula:

For a base of 2^k there is a cycle similar to the above of length 2(k+1)

The main application of the Reverse Then Add Algorithm in this case is to test the programming skills of computer scientists. Although it is pretty obvious that there are no palindromes for 196, computer scientists attempt to come up with better algorithms for the reverse then add algorithm, testing them on the ultimate number: 196.

Bibliography

http://mathworld.wolfram.com/Reverse-Then-AddSequence.html
From this website we got a lot of information on the Reverse Then Add Sum Process.

http://www.p196.org
This website is the main webpage for the computer programmers who have been doing the iterations of the Lychrel numbers. It had a lot of records which helped my understanding. This is also where we got the information about seed, kin, and thread numbers.

http://www.mathpages.com/home/kmath004.htm
This webpage had tons of information on the 196 Algorithm, the Reverse-Add Sum Process, Lychrel Numbers, Palindromic Number Conjecture, RATS Sequence, and more. This was where we got most of the mathematical information.

http://en.wikipedia.org/wiki/Lychrel_number
This is where we got historical and person information on the men who have been working on the 196 Palindrome, and information about Lychrel numbers.

<http://mathforum.org/library/drmath/view/51508.html>
This page contains a wealth of information not to be limited to the proof outlined above. It also is a source of cycles in other "power of two" bases.