3.) Let $a \in \mathbb{R}$ be given. Let $E = \{f > a\} = f^{-1}((a,\infty))$, then $E \in \mathcal{M}$ 3

Then $\{x : f(x+h) > a\} = \{x : x+h \in f^{-1}((a,\infty))\} = \{x : x+h \in E\}$ 3

$\{x : g(x) > a\}$ But $x+h = y \in E$ iff $x = y - h$ iff $x \in E - h$.

for some $y \in E$

So $\{x : g(x) > a\} = E - h$. 3 But by previous result, $E \in \mathcal{M} \Rightarrow E - h \in \mathcal{M}$. 3

So $\{g > a\} \in \mathcal{M}$, so $g \in \mathcal{M}(E)$. 3

4.) Since $f_{k+1}(x) \le f_k(x)$ for all k and all x, then

$f_{k+1}(x) > 1 \Rightarrow f_k(x) > 1$. So $E_{k+1} \subseteq E_k$

5

Thus $E_1 \supseteq E_2 \supseteq E_3 \supseteq \dots$

also we have $E = \bigcap_{k=1}^{\infty} E_k = \emptyset$, because for every $x \in E$, 5

we have $\lim f_k(x) = 0$, so $f_k(x) \le 1$ for some k, so $x \notin E_k$ for some k.

Also $|E_1| \le |E| < \infty$. 1 So by a result from class

$\lim |E_k| = |E| = |\emptyset| = 0$. 4

5.) Let $x \in G$ be given. Let $Z = \{x : f(x) \ne g(x)\}$, then $|Z| = 0$. 3

Let $k \in \mathbb{N}$ be given, then $(x - \frac{1}{k}, x + \frac{1}{k}) \cap G$ is open, so there

exists $\varepsilon > 0$ s.t. $(x - \varepsilon, x + \varepsilon) \subseteq (x - \frac{1}{k}, x + \frac{1}{k}) \cap G$. 3 Now 3

$(x - \varepsilon, x + \varepsilon) \subseteq Z$ is impossible since then we would have $|Z| > 2\varepsilon > 0$,

So there exists some point x_k in $(x - \varepsilon, x + \varepsilon)$ which is not in Z.

So $f(x_k) = g(x_k)$ 3 and $|x_k - x| < \frac{1}{k}$. Thus $\lim_{k \to \infty} x_k = x$ and

$f(x) = \lim f(x_k) = \lim g(x_k) = g(x)$. 3

6.) a) Since $\frac{1}{a} > 1$, then $\frac{1}{a}|E| > |E|$, so we can find $\varepsilon > 0$ s.t.

$\frac{1}{a}|E| > |E| + \varepsilon$. 4 Since $E \in \mathcal{M}$, we can find G open s.t. $E \subseteq G$ and

$|G| < |E| + \varepsilon$. 4 Write $G = \bigcup I_k$ 4 where I_k are open disjoint intervals.

Then $\sum |I_k| = |G| < |E| + \varepsilon < \frac{1}{a}|E|$.

4

b) Suppose not. Let $\{I_k\}$ be as in part a. Then $|I_k| \ge \frac{1}{a}|E \cap I_k|$ for all k, 4

So $\sum |I_k| \ge \frac{1}{a} \sum |E \cap I_k| = \frac{1}{a}|E \cap (\bigcup I_k)| = \frac{1}{a}|E \cap G| = \frac{1}{a}|E|$, which contradicts a.)