# Math 4433
## Test 3

For these problems, unless specifically requested otherwise, you may use without proof any result from class.

**1.** *(15 points)* Suppose $g : A \to \mathbf{R}$ and $f : B \to \mathbf{R}$, and $g(x) \in B$ for every $x \in A$, so that $f \circ g : A \to \mathbf{R}$. Let $c \in A$, and suppose $g$ is continuous at $c$ and $f$ is continuous at $g(c)$. Show that $f \circ g$ is continuous at $c$.

**2.** *(15 points)* State carefully and prove the product rule for derivatives.

**3.** *(10 points)* Suppose $f : [0,1] \to \mathbf{R}$ is such that $f(1/n) = (-1)^n$ for all $n \in \mathbf{N}$. Prove that $\lim_{x\to 0} f(x)$ cannot exist.

**4.** *(10 points)* Suppose $f : [0,1] \to \mathbf{R}$, and there exists a sequence $x_n$ such that $0 \le x_n \le 1$ for all $n \in \mathbf{N}$, and $f(x_n) = n$ for all $n \in [0,1]$. Show that $f$ cannot be continuous on $[0,1]$.

**5.** *(20 points)* Consider the following two statements, one of which is true and one of which is false:

(i) If $f : [0,1] \to \mathbf{R}$ is continuous at 0, and $f(1/n) > 0$ for all $n \in \mathbf{N}$, then $f(0) > 0$.

(ii) If $f : [0,1] \to \mathbf{R}$ is continuous at 0, and $f(1/n) \ge 0$ for all $n \in \mathbf{N}$, then $f(0) \ge 0$.

**a.** Identify which of the two statements is true, and prove it.

**b.** Give an example of a function showing that the other statement is false.

**6.** *(15 points)* Prove that the equation $10^x = 2$ has a solution. (You may assume that $10^x$ is a continuous function on $\mathbf{R}$.)

**7.** *(15 points)* Suppose $g(x)$ is bounded on $[0,1]$, and let $f : [0,1] \to \mathbf{R}$ be defined by $f(x) = x^2 g(x)$. Show that $f$ is differentiable at 0, and find $f'(0)$.