Wasan and *Sangaku*
Japanese Temple Geometry

By

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Throughout the Japanese Tokugawa period from 1600-1868, the country of Japan practiced a strict habit of self isolation. During this era all contact with foreign influences was rigorously prohibited, except for a few contacts with Dutch traders in limited regions. The lack of relations with others countries allowed Japanese culture to flourish. One result of this isolation is *Wasan,* or "Japanese mathematics." Created separately from western European mathematics, there are still similarities between the different forms: Japanese *wasan* still contemplates a great deal of Euclidean geometry. However, there are some predominate differences in Japanese geometry, mainly that great emphasis is placed on the study of close packing of circles into larger figures. Even still, some similarities can be seen between *wasan* schools and the Pythagoreans: both developments discuss the profound relationship between math and art. *Wasan* is an integral part of Japanese culture and offers some insight into Japanese mathematics, religion, and art.

A particular feature of *wasan* geometry that highlights the cultural association between numbers and art is *sangaku,* which literally means "mathematical tablet". A common practice during the Tokugawa administration was to offer geometric problems and solutions inscribed on wood tablets to the gods. The custom of hanging tablets at shrines was established in Japan centuries before *sangaku* was developed; however some of the oldest surviving tablets of *sangaku* date from 1683. Devotes of math, most likely samurai, farmers, and merchants would solve an assortment of geometry problems and present them on delicately colored wooden tablets which would then be hung under the roofs of religious buildings. Usually only the result of the theorem was given, not the proof. Admirers could enjoy the beauty of these tablets or could attempt to solve a problem themselves.

In 1868 the Tokugawa regime ended and was replaced by the Meiji period during which the country of Japan opened up to influences from other countries. Since *wasan* had never been used to describe any phenomena of nature, it seemed unpractical to continue the tradition. 1872 – Ministry of Education orders that state schools stop teaching wasan a decade later, it had slipped into obscurity

Japanese math only described static systems, not dynamic ones, so the first translations of western works were done using Japanese philosophical terms instead of *wasan* terms. Thus western math practices, which were being applied to things like physics, mechanics and other scientific realms, were adopted in Japan.

Sangaku Presentation Bibliography
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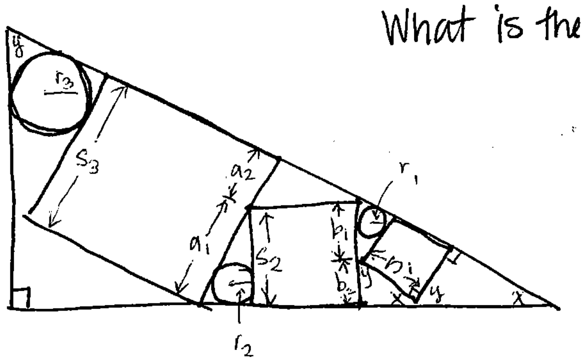
___. "Incenter"

___. "Incircle."

OUTLINE

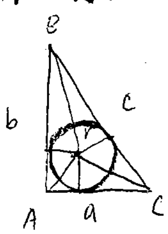
1. **Tokugawa Regime in Japan**
    a) self-isolation 1600-1868
    b) culture flourishes
    c) wasan created
2. **Wasan**
    a) different than Western math
    b) Euclidean geometry
    c) Emphasis on spheres and circles
    d) Wasan schools similar to Pythagorean schools
3. **Sangaku "mathematical tablet"**
    a) division of wasan
    b) reflective of Japanese culture
    c) art, religion, and mathematics
    d) merchants, farmers, samurai class
    e) geometric theorems inscribed on wooden tablets
    f) colorful and detailed
    g) show result, not proof
    h) hang from the roofs of religious buildings
    i) admirers
4. **Proof of triangle problem**
5. **Proof of Casey's theorem**

What is the relationship of the radii of the circles?



First observe that all of the inner triangles are similar to each other. If we label one triangle with angles x, y, and L (right ∠) then by complementary angles it is easy to label all angles of every triangle, including the biggest one.

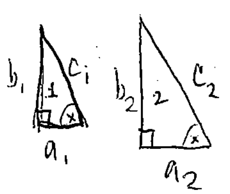
Second we observe that each triangle ~~can~~ area can be expressed interms of the radius of the incircle, and from this we find that if the triangles are similar by some ratio k, then the radii are also similar by the ratio k.



Each triangle can be expressed as the sum of 3 smaller triangles (the area can be expressed) that are made of one side of the large Δ and each have a height r. Also, r is then the radius of the "incircle" of the large triangle.

So area of $\Delta ABC = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr = \frac{1}{2}r(a+b+c)$. Let $S = \frac{a+b+c}{2}$ so Area $\Delta ABC = r_i S$ where r_i is the radius of the incircle of Δ_i.

Take two right triangles (since we are only dealing w/right triangles in this problem) → that are similar



the areas equal $\frac{1}{2}a_i b_i$ and

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = K$ so $a_2 = Ka_1$ and $b_2 = kb_2$ so Area $\Delta 2 = \frac{1}{2}ka_1kb_1 = \frac{1}{2}k^2a_1b_1$

$\# k^2$

Now Area (A_i) of triangle 2: $A_2 = K^2A_1 = r_2S_2$

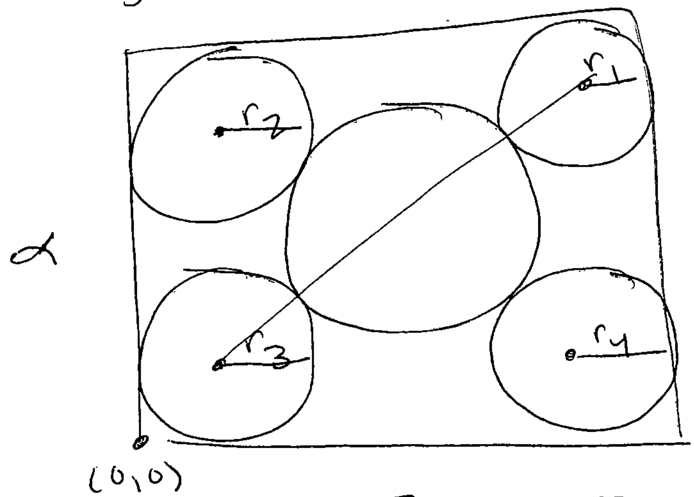
$k^2 r_1 s_1 = r_2 k s_1$

$kr_1 = r_2$

$k = \frac{r_2}{r_1}$

So the radii are comparable by some ratio k that is the same ratio for comparing the triangles.

Casey's Theorem



What is the relationship between r_1, r_2, r_3, r_4 and α?

Casey's Thm: $T_{12}T_{34} - T_{13}T_{42} + T_{14}T_{23} = 0$

obviously, 4 of the 6 T's are:

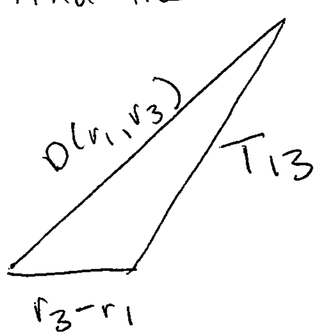
$$T_{23} = \alpha - r_2 - r_3$$

$$T_{14} = \alpha - r_1 - r_4$$

$$T_{12} = \alpha - r_1 - r_2$$

$$T_{34} = \alpha - r_3 - r_4$$

To find the other two, we use the Pythagorean theorem:



ALOG $r_3 > r_1$.

$$T_{13}^2 + (r_3 - r_1)^2 = (\alpha - r_1 - r_3)^2 + (\alpha - r_1 - r_3)^2$$

$$T_{13} = \sqrt{2(\alpha - r_1 - r_3)^2 - (r_3 - r_1)^2}$$

Similary, $T_{24} = \sqrt{2(\alpha - r_2 - r_4)^2 + (r_4 - r_2)^2}$ if we ALOG $r_4 > r_2$.