

## Review for First Exam

The first exam will cover sections 1.2, 2.1, 2.2, 2.3, 2.4, 3.1, 3.2, and 3.3 of the text. The relevant assignments are assignments 1 through 10.

There will be one or two questions in which I ask you to state a definition or prove a theorem. Here is a list of the definitions and proofs which might appear, with references to where you can find them in the text. Sometimes I gave a proof in class which is different from the one in the class; you can use whichever one you prefer.

- Definition of upper bound and lower bound (2.3.1).
- Definition of supremum and infimum (2.3.2).
- Be able to state and prove the Archimedean property of real numbers (2.4.3).
- Definition of sequence (3.1.1).
- Definition of limit of a sequence (3.1.3).
- Be able to prove that a sequence can have at most one limit (3.1.4).
- Be able to prove that a convergent sequence is bounded (3.2.2).
- Be able to prove that if  $(x_n)$  converges to  $x$  and  $(y_n)$  converges to  $y$ , then  $(x_n + y_n)$  converges to  $x + y$ . This is part of (3.2.3); I won't ask for proofs of the other parts of (3.2.3) on this exam.
- Be able to prove that if  $(x_n)$  converges to  $x$  and  $x_n \geq 0$  for all  $n \in \mathbf{N}$ , then  $x \geq 0$  (3.2.4).
- Definition of increasing and decreasing sequence (3.3.1).
- Be able to prove that if  $(x_n)$  is a bounded increasing sequence, then  $(x_n)$  converges (3.3.2(a)).

The rest of the exam will consist of questions similar to the homework problems. Here is a guide to the sections in the text that will be covered on the exam.

- Section 1.2: you should be familiar with the well-ordering property of  $\mathbf{N}$  and the principle of mathematical induction. If you're rusty on proofs by induction, there are some good examples in this section.
- Section 2.1: you should have some idea of what the algebraic and order properties of  $\mathbf{R}$  are, if only to know what things you can assume to be self-evident in your proofs. Notice also that Bernoulli's inequality is stated and proved in this section. You do not need to read the subsection titled "Rational and Irrational Numbers".
- Section 2.3: the whole section is worth reading.
- Section 2.4: for now, you do not need to read the subsections titled "Functions", "The Existence of  $\sqrt{2}$ ", or "Density of Rational Numbers in  $\mathbf{R}$ ".
- Section 3.1: the whole section is worth reading. There is a little bit in this section we didn't cover in class, but which is still useful to know.
- Section 3.2: the whole section is worth reading. I won't expect you to know or use Theorem 3.2.11, but it certainly doesn't hurt to know it.
- Section 3.3: the whole section is worth reading.