Math 4163
Review for Exam 1

The first exam will cover sections 2.1, 2.2, all of 2.3, 2.4.1, and 2.5.1 of the text. There are some suggestions below as to how to review the text. You should also go over the two homework assignments and the quiz, and maybe try some of the following additional problems from the supplementary problem sheet: 2.2, 2.3, 3.1, 3.5, 3.6. Note: these problems are available on the course web page — click on the link titled "Supplementary Problem Sheet". Hopefully the lecture notes will also be of some use!

2.1. Introduction. This section just states the problem whose solution is explained in section 2.3. Actually, in section 2.3, we only consider the case when the temperatures $T_1(t)$ and $T_2(t)$ are zero, and the "source term" $Q(x,t)$ is zero, so you don't need to worry about those till later, after this exam. See section 2.3.1.

2.2. Linearity. There is something in this section you should pay attention to, not because it helps you do any calculations, but because if you don't know it, it's hard to understand what's going in the rest of the chapter. It's the "Principle of Superposition" in the box on page 37. What it says is that, for the equations we consider in this chapter, if you can find several solutions of the equation, then you can find many more new solutions by taking combinations of the one you've already found. For example, if we know that $\cos(2x)e^{-20t}$ and $\cos(3x)e^{-45t}$ are solutions of the heat equation for Dirichlet boundary conditions on some interval, then it immediately follows that anything of the form $C_1\cos(2x)e^{-20t} + C_2\cos(3x)e^{-45t}$ is also a solution. The reason that's important is that it underlies the whole idea of the separation of variables, which is to obtain the desired solution of a P.D.E. by first finding special "separated" solutions, none of which are necessarily the solution we're looking for, and then finding the desired solution as a combination of the separated solutions.

2.3. Heat equation with zero temperatures at finite ends.

2.3.1. Introduction. As we go through this course, you'll want to keep track of the different problems we look at, and how they differ from each other. In particular, you'll want to become familiar with why each problem is given the name that it has. Why is this section referring to "finite ends"? It's to distinguish the problem solved here from the one solved in Chapter 10, where the heat equation on an infinitely long rod is considered. By the way, the problem introduced here is the one that I called the "Dirichlet problem for the heat equation" in class.

2.3.2. Separation of variables. The material in this section is crucial, as you've gathered already by the fact that I've repeated it in class several times. To really get a grasp on it, though, you have to work new problems and struggle through them. Now that you've taken the first quiz and done problem 2 on it, you probably appreciated that equation (2.3.7) is not as innocuous as it seems. Once we make the decision to put the k in with the G's instead of the ϕ's, and to call the constant $-\lambda$ instead of λ, we have to deal with the consequences. This same issue becomes important in solving problem 3.4 on the supplementary problem sheet (I worked this one out in class one day).

2.3.3. Time-dependent equation. You really need to read only to (2.3.13) in this section, and can stop there.

2.3.4. Boundary-value problem. You should be quite familiar with all the contents of this section (except you can skip the paragraph on page 47 about springs and masses). In particular, it makes this class much easier if you have a fluent understanding of the words "eigenvalue" and "eigenfunction". By fluent understanding, I mean that you should know and understand the definitions of these terms. Don't just think of an eigenvalue and eigenfunction as something you find at the end of a long, mysterious, process — that misses the point. Instead, think of them in these terms: An eigenvalue λ is a number associated with a certain boundary-value problem for the equation $d^2\phi/dx^2 = -\lambda\phi$. It is defined as a number for which the boundary-value problem has a *non-trivial* solution (i.e., a solution which is not the constant function $\phi \equiv 0$.) Different boundary-value problems therefore have different eigenvalues — each boundary-value problem has its own particular set of eigenvalues. (This is, in fact, the origin of the term “eigenvalues”, which is German for “particular values”.) For example, the Dirichlet problem on $0 \leq x \leq 5$ has the eigenvalues $(n\pi/5)^2$, for $n = 1, 2, 3, \ldots$; whereas the problem on question 1 of the Quiz had the eigenvalues $n^2/4$ for $n = 1, 3, 5, 7, \ldots$. An eigenfunction is just a non-trivial solution that goes with a particular eigenvalue. So each eigenvalue has its own eigenfunctions (in general, a given eigenvalue can have more than one eigenfunction).

2.3.5. Product solutions and the principle of superposition. Again, you should read and understand this section completely. It contains the main idea of the method of separation of variables.

2.3.6. Orthogonality of sines. The object of this section is to explain how to find the numbers B_n in equation (2.3.31), once you are given the function $f(x)$. To do this, you multiply (2.2.31) by cos $\frac{m\pi x}{L}$ and use equation (2.3.32). You should read the entire section. The word “orthogonal” defined in the last paragraph is a useful vocabulary term we'll be using frequently later.

2.3.7. Formulation, solution, and interpretation of an example. This section is actually just a worked-out homework exercise. You can consider it as an explanation of the solution of a problem which is just like problem 3.1 on the supplementary problem sheet. I suppose all of you can read this section easily. You can expect to have a problem like this on the exam.

Appendix to 2.3: Orthogonality of functions. You can skip this section entirely. If you're curious, it attempts to explain why what we call “orthogonality” in this book has anything to do with the orthogonality you learned about when studying vectors.

2.4. Worked examples with the heat equation (other boundary-value problems).

2.4.1. Heat conduction in a rod with insulated ends. This section introduces what I called the Neumann problem for the heat equation. The idea is that when you solve this problem by the method of separation of variables, you have to use different eigenvalues and eigenfunctions than you did for the Dirichlet problem. Read this section in its entirety.

2.4.2. Heat conduction in a thin circular ring. Skip this section. We will cover it next week, after the exam.

2.4.3. Summary of boundary-value problems. Look at the first two columns of table 2.4.1 on page 69. To understand the entries in the rows labeled “Eigenvalues and eigenfunctions”, compare and contrast the procedure done between (2.3.15) and (2.3.21) with that done between (2.4.9) and (2.4.15). To understand the entries in the rows labeled “Series” and “Coefficients”, compare and contrast the procedure done between (2.3.31) and (2.3.35) with that done between (2.4.21) and (2.4.24). Ideally, once you've done that, you might start to think that you could add more columns to the table on your own. I won't ask for such an ambitious undertaking on the test, but we will be filling in the third column soon afterwards.

2.5. Laplace's equation: solutions and qualitative properties.

2.5.1. Laplace's equation inside a rectangle. If you like, you can start reading this section from equation (2.5.7) onwards. We covered the material from there to the end of the section. See also exercise 2.5.1 at the end of the section.