Review for Second Exam

The second exam will cover most of sections 3.1 through 3.8 of the text.

3.1 You should have already covered the material in this section in Calculus III.

3.2 In class we discussed Definition 3.4 and went over the proofs of some of the parts of Theorem 3.2. Most of the time in this class the vector spaces we look at will either be R^n (the space of all $n \times 1$ column vectors), R_n (the space of all $1 \times n$ row vectors) or subspaces of R^n or R_n.

A vector space is any set of objects which can be added and multiplied by scalars and which satisfy the properties (a), (b), and (1) through (8) of Definition 3.4. We call the objects in a vector space "vectors" even though they don't necessarily look like the vectors you are familiar with from Calculus III. Thus a vector could be a matrix, or a polynomial. The important thing is that they can be added and multiplied by scalars in the usual way.

The book uses, at first, the symbols $\oplus$ and $\odot$ for addition and scalar multiplication of vectors, but then drops these symbols after a while and uses the usual $+$ and $\cdot$ symbols (or no symbol at all for scalar multiplication). The reason for using the funny symbols at first is to remind you that addition and scalar multiplication don't have to be defined in the usual way that you learned in Calculus III — just as vectors don't have to look like the vectors you used in Calculus III — all that's important is that they satisfy the usual properties. The book illustrates this point with several examples, but for us the main examples would be Examples 1, 2, 4, and 5.

3.3 We covered all of this section, except for the last subsection titled "Lines in R^3", which is review from Calculus III. You should read and understand all of the Examples in this section. Pay special attention to Theorem 3.2, Definitions 3.6 and 3.7, and Example 10 (which contains the definitions of the term "solution space" and "null space").

3.4 This section, together with section 3.5, are the most important sections in the book, and in particular, understanding Definition 3.9 is the key to understanding the material which comes later in the course.

You should try to read and understand everything in the section. If you don't have time for that, or are trying but not succeeding in understanding something, plan to come back to the section later on (in a couple of weeks, say) and re-read it. If you're particularly short on time you could probably get away with skipping most of the material from Theorem 3.5 to the end of the section, for now.

3.5 You should read Definition 3.10 and Examples 1, 2, and 3. You should also read Theorem 3.7 and its proof.

You can skip Example 4, and all the material on pages 168 to 170, except that you should read, and be able to use, the procedure given in the blue box at the bottom of page 170 for finding a subset of $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ that is a basis for the subspace W spanned by S. NOTE: in the examples given in the text, $W = \text{span}\ \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is usually all of $\mathbf{R}^n$

or all of $\mathbf{R}_n$, but this is not always the case. See problem 11 on page 177 and problem 2 on Quiz 2.

Read Example 5 and the Remark which follows.

I proved Theorem 3.9 and Corollary 3.1 in class, but I gave a different proof from the one that appears in the book, so if you understood the proof I gave in class you can skip reading this Theorem and Corollary.

Read Definition 3.11, Example 6, and Example 7. You can skip the rest of the section, if you like.

3.6 In this section, you can skip the first couple of pages (which is just a rehash of something you already know how to do), and start reading from page 181 with the paragraph that begins "The procedure for finding a basis for the solution space of a homogeneous system ... ". You can just read from there through Example 1 and then skip the rest of the section.

3.7 Read from the beginning of the section through Example 2. Skip the subsection on Isomorphisms, and read the subsection on Transition Matrices from its beginning on p. 193 to the end of Example 3 at the top of p. 195. You can skip the material on pp. 196 – 198 if you like, although I did notice from class discussions that some students had read it and found it instructive.

3.8 Read from the beginning of the section through Example 5. Example 5 is important. Also read the statements of Theorem 3.17 and Theorem 3.18. You don't have to read the proof of Theorem 3.17. You can skip the material on pages 208 – 211, it won't be covered on this exam. The blue box on p. 211 is worth looking at, but remember it only applies to square $n \times n$ matrices.