Review for Second Exam

The second exam will cover sections 1.5, 2.5, 3.1, 3.2, and 3.3 of the text. The relevant assignments are Assignments 4, 5, and 6.

1.5. This section re-does the material from sections 1.1 and 1.2 in higher dimensions. You should already know from sections 1.1 and 1.2 the meanings of the variables u, e, ρ, ϕ, c, K_0, and k, along with the correct physical units for each one. The main difference in higher dimensions is that now instead of ϕ, which was a number, we consider the vector $\vec{\phi}$.

In section 1.2, ϕ was defined as the rate at which heat energy flows per unit area per second in the positive x-direction. Now, in section 1.5, we define $\vec{\phi}$ as a vector which can be used to find the rate of heat flow in any direction. Namely, to find the rate of heat flow (in Joules per unit area per second) in the direction parallel to a unit vector $\mathbf{n}$, we take the dot product of $\mathbf{n}$ with $\vec{\phi}$.

You should remember the conservation of energy equation, which is given in the text in three different, equivalent forms: namely equations (1.5.1), (1.5.4), and (1.5.6). Equation (1.5.1) is pretty easy to remember, because it just says that the rate of change of heat energy within a region equals the rate at which heat flows into the region plus the rate at which heat is created by heat sources. The only difference between (1.5.1) and (1.5.4) is that the double integral on the right-hand side of (1.5.1) is replaced by a triple integral in (1.5.4); you recall from Calculus IV that according to the Divergence Theorem, these two integrals are equal. Finally, (1.5.6) just says that the quantity being integrated in (1.5.4) is everywhere zero.

You should also remember Fourier's law of heat conduction, (1.5.7). It is just like the one-dimensional law (1.2.8), except now the derivative of u with respect to x is replaced by the gradient of u. Again, you recall from Calculus IV that the gradient of u is the vector given by

$$\nabla u = \frac{\partial u}{\partial x}\,\mathbf{i} + \frac{\partial u}{\partial y}\,\mathbf{j} + \frac{\partial u}{\partial z}\,\mathbf{k}.$$

Finally, combining Fourier's law with the law of conservation of energy gives the heat equation (1.5.8). Note that what the book calls $\nabla \cdot \nabla$, or ∇^2, I sometimes denoted in class by the symbol Δ. See equation (1.5.10).

Typical boundary conditions for the heat equation in higher dimensions are that the temperature is held constant on the boundary, say $u = 0$ on the boundary; or that there is no heat flow through the boundary, i.e., $\vec{\phi} \cdot n = 0$ or $\nabla u \cdot \mathbf{n} = 0$ on the boundary; or that u is held constant on some parts of the boundary and there is no heat flow on other parts of the boundary.

We have not yet solved the time-dependent heat equation in higher dimensions in this class (in the text, that is not done till chapter 7). Instead, we've only considered cases in which the temperature is independent of time, in what is called equilibrium temperature distributions. In that case, the temperature satisfies the equation $\nabla^2 = 0$, which is called Laplace's equation.

You should review the paragraphs titled "Two-dimensional problems" and "Polar and cylindrical coordinates" on pages 27 and 28. In class I spent a good deal of time deriving equation (1.5.19), so I hope you are familiar with it by now, but you do not need to memorize this equation. If I ask any question requiring the use of that equation, I'll provide it to you.

You can skip the paragraph titled "Spherical coordinates".

2.5. You should have read and understood all of sections 2.5.1 and 2.5.2. You can skip 2.5.3 completely. From section 2.5.4 you should read the paragraphs titled "Mean value theorem" and "Solvability condition". The material in the latter paragraph was used in a couple of homework problems already. The mean value theorem was covered in class; it hasn't been used in a homework problem yet, but it is simple and often useful.

3.1. This section defines the notions of "piecewise smooth function" and "periodic extension of a function". Actually, the definition of "piecewise smooth" given here was a little too vague for my taste, so I went into more detail in the lecture notes. You might re-read the lecture notes if you want to know more exactly what is meant by "piecewise smooth".

3.2, 3.3. You should read all of these sections, except that you can skip subsection 3.3.4 ("Even and odd parts"), which won't be needed for the exam.

As I mentioned in class, I think it's a good idea to actually memorize a couple of basic Fourier series. The ones that seem to be most basic are (1) the Fourier sine series for the function $f(x) = 1$ on $[0, L]$ (this is the series in (3.3.10), divided by 100), which is also the Fourier series for the odd extension of this function on $[-L, L]$ (see Figure 3.3.3); and (2) the Fourier sine series for $f(x) = x$ on $[0, L]$, which is also the Fourier series for $f(x) = x$ on $[-L, L]$ (see p. 104). The example on pp. 107-108 (Fourier cosine series for $f(x) = x$, or Fourier series for the "sawtooth" function) is also pretty basic.

You should go over the theorems enclosed in boxes on pages 92, 111, 112, and 113, and interpret them in terms of the examples mentioned in the above paragraphs.