Review for Second Exam

The second exam will cover sections 3.4, 3.5, 3.7, 4.1, and 4.2 of the text. The relevant assignments are assignments 12 through 17.

As did the first exam, the second exam will contain one or two questions in which I ask you to state a definition or prove a theorem. Here is a list of the definitions and proofs which might appear, with references to where you can find them in the text. As usual, if the proof I gave a proof in class is different from the one in the text, you can use whichever one you prefer.

- Definition of subsequence (3.4.1).
- Be able to prove that if a sequence converges to x then so does any of its subsequences (3.4.2).
- You should know the statement of the Bolzano-Weierstrass Theorem (3.4.8), but I will not ask for its proof on the exam.
- Definition of Cauchy sequence (3.5.1).
- Be able to prove that if a sequence converges, then it is Cauchy (3.5.3). You should also be aware that the converse is true (if a sequence is Cauchy, then it converges), but I will not ask for a proof of the converse on the exam.
- Definition of infinite series (3.7.1). You can give this definition, if you like: An infinite series Σx_n is a sequence (s_n) given by $s_1 = x_1$ and $s_{n+1} = s_n + x_{n+1}$ for all $n \in \mathbf{N}$.
- Be able to prove the comparison test for series (3.7.7). Note: the proof I gave in class is quite different than the one given in the text.
- Definition of limit of a function. This is given in (4.1.4), but in class we work with a slightly different definition; namely: a real number L is said to be a limit of f at c if, given any $\epsilon > 0$, there exists $\delta > 0$ such that if $0 < |x - c| < \delta$ then $f(x)$ is defined and $|f(x) - L| < \epsilon$.
- Be able to prove that if $\lim_{x \to c} f(x) = L$, then for every sequence (x_n) such that x_n converges to c and $x_n \neq c$ for all $n \in \mathbf{N}$, the sequence $(f(x_n))$ converges to L. Note that this is the implication (i) $\Longrightarrow$ (ii) in Theorem 4.1.8 of the text (Sequential Criterion for Limits). I will not ask for the proof of the reverse implication, (ii) $\Longrightarrow$ (i).

The rest of the exam will consist of questions similar to the homework problems. Here is a guide to the sections in the text that will be covered on the exam.

- Section 3.4: This whole section is relevant to the exam.
- Section 3.5: Read from the beginning of the section through the examples in 3.5.6; we did not cover the material on contractive sequences (3.5.7 and beyond).
- Section 3.7: It's worth reading the whole section, but you can skip the Limit Comparison Test (3.7.8) if you wish.
- Section 4.1: You do not need to read the material on cluster points (4.1.1, 4.1.2, and 4.1.3), since the definition of limit we used in class does not refer to cluster points. The remainder of this section is relevant.
- Section 4.2: This whole section is worth reading (and is not hard to read).