

## Review for Final Exam

The final exam is comprehensive, covering the material from all the assignments (1 to 9), and all the sections of the book we've covered so far (1.1 through 1.5, 2.1 through 2.5, 3.1 through 3.5, and 4.1 through 4.4). To review for it, you can use the review sheets from the first two exams (available on the course web page), together with this sheet, which covers material we have had in class since the second exam.

**3.4.** Review the entire section. The main theorems are those given in the second and fourth boxes on p. 118 and in the second box on p. 120. You should understand the proof on pages 120-121; I gave a somewhat more concise version in class. A similar proof was assigned for homework (problem 3.4.2). The method of eigenfunction expansion is also important, it was used in two problems on Assignment 7 (problems 3.4.11 and 3.4.12). Again, in reviewing this material it's very helpful to keep a couple of simple Fourier series in mind (like the Fourier sine series for  $f(x) = 1$ , or the Fourier cosine series for  $f(x) = x$ , or the Fourier sine series for  $f(x) = x$ ), and go over what happens when you apply the theorems or techniques to these series.

**3.5.** All you might need to know from this section would be the statement in the box on p. 127; namely, that the Fourier series of a piecewise smooth function  $f$  can be integrated term-by-term to give the Fourier series for the integral of  $f$ . Thus, for example, if you know the Fourier series for the function  $f(x) = x$  on  $[-L, L]$ , it's easy to find the Fourier series for  $x^2/2$  on  $[-L, L]$ . All you have to do is integrate each term in the Fourier series for  $x$ .

**4.2** It's certainly helpful to understand the material in this section, but I won't test you specifically on this material.

**4.3.** In class, we discussed the wave equation on  $0 \leq x \leq L$  with boundary conditions

$$u(0, t) = 0 \quad \text{and} \quad u(L, t) = 0$$

for  $t > 0$ . This section discusses physical conditions on a vibration string under which other boundary conditions might be appropriate. For the test, you won't need to know this.

However, for reviewing the method of separation of variables, it would be a good exercise, to think about how to solve the wave equation with other boundary conditions. What would happen, for example, if we solved the wave equation on  $0 \leq x \leq L$  with the boundary conditions

$$\frac{\partial u}{\partial x}(0, t) = 0 \quad \text{and} \quad \frac{\partial u}{\partial x}(L, t) = 0$$

for  $t > 0$ ?

**4.4** You should review the entire section. The answers to problems 4.4.2(c) and 4.4.3(b) have been posted online.