

Math 4433 Review for Final Exam

The final exam, which is comprehensive, will cover the following sections of the text: 2.1 through 2.4, 3.1 through 3.5, 3.7, 4.1, 4.2, 5.1 through 5.3, 6.1, and 6.2.

Here are the definitions and proofs which I might ask for:

- Definition of supremum and infimum (2.3.2).
- Completeness property (also called “completeness axiom”) (2.3.6).
- Definition of limit of a sequence (3.1.3).
- Be able to prove that a sequence can have at most one limit (3.1.4).
- Be able to prove that if (x_n) converges to x and (y_n) converges to y , then (i) $(x_n + y_n)$ converges to $x + y$ and (ii) $(x_n y_n)$ converges to xy .
- Definition of a Cauchy sequence (3.5.1).
- Be able to prove that if X is a convergent sequence, then X is Cauchy (3.5.3).
- Definition of infinite series (given in class, or see (3.7.1)).
- Be able to prove the n th Term Test (3.7.3): if $\sum x_n$ converges then $\lim(x_n) = 0$.
- Definition of limit of a function at a point (4.1.4).
- Definition of function being continuous at a point (you can give either (5.1.1) or the shorter definition I gave in class).
- Definition of derivative (you can give either (6.1.1) or the shorter definition I gave in class).
- Proof of the sum rule (6.1.3(b)) and product rule (6.1.3(c)) for derivatives.

As a guide to reviewing the sections from 2.1 through 4.1, you can use the review sheets for the first two exams, both of which are available at the course web site. Here is a guide to reviewing the remaining sections that will be covered on the exam.

- Sections 4.2, 5.1, and 5.2: We covered all the topics in these sections in detail, so it’s worth reviewing as much of it as you can. In particular, each of the examples is a mini-proof in itself which you should study carefully. Try to compare and contrast examples between the different sections. For instance, notice that the same functions $f(x) = x \sin(1/x)$ and $f(x) = \sin(1/x)$ are used as examples in several different places.
- Section 5.3: We only covered this section in detail up through Theorem 5.3.4. You need not review the material which comes after that.
- Section 6.1: We only covered the material in this section up to (but not including) the subsection on the Chain Rule (p. 161). It certainly wouldn’t hurt to review the Chain Rule (6.1.6) and the examples in 6.1.7, though. You won’t need to use the Chain Rule on the exam, but reading about it helps put other things in perspective. In particular, example 6.1.7(e) relates to similar examples in chapters 4 and 5. On the other hand, you can safely skip the material on inverse functions (pp. 164 – 166) completely.
- Section 6.2: From this section you should just know the statements of Rolle’s theorem (6.2.2) and the Mean Value Theorem (6.2.4) (you do not need to know their proofs). I may also ask one question on the Mean Value Theorem similar to problem 6.2.6 or 6.2.13. A good way to prepare for that would be to read the proofs of Theorems 6.2.5 and 6.2.7 (they are short), and examples 6.2.10(a) and 6.2.10(b).