Review for Final Exam

The final exam will cover all the sections of the text covered on the first two exams (5.4, 6.2, 6.4, 7.1, 7.2, 7.3, 8.1, and 8.2), together with sections 8.3, 8.4, 9.1, 9.2, 9.3, and 9.4. It will probably be weighted more towards the type of problems we've done in the latter half of the course.

The final will have a format similar to that of the first two exams, with a few questions at the beginning asking for definitions and statements and/or proofs of theorems. The list of definitions, statements of theorems, and proofs which I might ask for consists of the lists on the review sheets for the first two exams, together with the following:

- Know the definition of absolute convergence (9.1.1).
- Know the statements of the Root and Ratio Tests (9.2.3 and 9.2.5). I will not ask for a proof.
- Know the statement of the Alternating series test (9.3.1). I will not ask for a proof.
- Know the definitions of convergence, absolute convergence, and uniform convergence of a series of functions on a subset D of $\mathbf{R}$ (9.4.1).
- Know the statement of the Weierstrass M-test (9.4.6). I will not ask for a proof.

As a guide to reviewing for the exam, you can use the review sheets for the previous two exams, supplemented by the following comments.

- Sections 8.3 and 8.4: I gave this material a different treatment in the lectures than what you will find in the text. If you like, you can skip the treatment in the text entirely, and just review the lecture notes. Of course, it doesn't hurt to read from the text, and if you carefully compare what's in the text with what's in the lecture notes, you'll find the differences aren't so great. At any rate, the problems at the end of these sections make useful practice problems

- Section 9.1: You should read the whole section, except that you don't need to worry about the proofs of Theorems 9.1.3 and 9.1.5. The Theorems themselves are important, though.

  One tricky aspect of Theorem 9.1.3 is that it does NOT imply that if a series $\sum x_n$ is divergent, then any series obtained from it by grouping the terms is also divergent. In fact, that is not true, as is shown by the example of the series

  $$1 - 1 + 1\frac{1}{2} - 1 + 1\frac{1}{4} - 1 + 1\frac{1}{8} - 1 + 1\frac{1}{16} - 1 + \dots,$$

  which is divergent (do you see why?), and the series

  $$(1 - 1) + (1\frac{1}{2} - 1) + (1\frac{1}{4} - 1) + (1\frac{1}{8} - 1) + (1\frac{1}{16} - 1) + \dots,$$

  which is convergent. This question came up on problem 5 of section 9.3: see the solution to this problem posted at the course webpage.

- Section 9.2: In class I proved Corollary 9.2.3 and Corollary 9.2.5 by a slightly different method than what is used here (which didn't involve going through Theorem 9.2.2 or 9.2.4). It would be good review to re-read the proofs I gave in class; you don't need to read the book's version.

  I did not cover the Integral Test or Raabe's Test in class (except to briefly mention Raabe's test). You won't need to know them for the exam, but I suppose knowing them could come in handy (especially the Integral Test) if you're wrestling with the question of whether a given series converges. In particular, one of the consequences of the integral test is that the series $\sum 1/n^p$ converges if and only if $p > 1$; this is often useful to know.

- Section 9.3: We covered this whole section, except that we ran out of time before I could do Example 9.3.6, which is very interesting. Notice that it follows from Example 9.3.6 that $\sum \frac{\sin n}{n}$ is a convergent series: this would be quite hard to prove without Dirichlet's test.

- Section 9.4: We covered this whole section in class, except in a somewhat different way than in the text. Again, I think it would help things to gel in your mind if, in addition to reviewing the lecture notes, you read the book's treatment of the material and compared them.

  In the problems at the end of this section (and probably elsewhere as well), there are places where you almost have to use L'hopital's Rule. Although we skipped section 6.3, because I didn't want to spend time on the PROOF of L'hopital's rule, I still expect you to be able to use L'hopital's rule, based on what you learned in Calculus. If you're a little rusty on how to use it, you might want to go back and read examples 6.3.6 and 6.3.6 in section 6.3.